Hints and Solutions

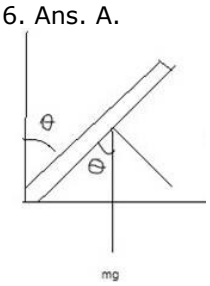
1. Ans. A.
Stress=Force (F)/Area (A)
Force=mg and also Density (d) =mass (m)/volume (V), therefore m=density (d) × volume(V)
Stress=mg/A=dVg/A,
Let the linear dimension be L, then the Volume will be, $V=L^3$ and Area will be, $A=L^2$
So, Stress=dL^3g/L^2
Stress=dLg
Stress $\propto$ L, so stress will change by a factor of 9.

2. Ans. C.
According to the equation of motion, the velocity at a time t is represented by:
v=u+at, where v is the final velocity, u is the initial velocity t is the time and a is the acceleration produced in the body. Since, the body is thrown upwards i.e. opposite to the direction of force of gravity so,
a=(-g), negative sign indicates that the body is thrown in the opposite direction of the force of gravity. The new equation will look like as following:
v=u+(-g)t
v=u-gt
So, the graph will be a straight line with negative slope.

3. Ans. C.
Initially, kinetic energy,$K_1=1/2mv_o^2$
After 10 seconds the kinetic energy becomes
$K_2=1/8mv_o^2$
Such that:
$1/2mv^2=1/8mv_o^2$ from this equation:-
$v^2/v_o^2=1/4$
$v=v_o/2=10ms^{-1}/2=5ms^{-1}$
it is given that the frictional force experienced by the body is:
$F=(-k)v^2$, (i)
Also F=ma=mdv/dt , by putting this value in equation (i) we get
$mdv/v^2=(-k)dt$
On integrating both the sides of the above equation we get:
$$m\int_{10}^{5} dv/v^2=(-k)\int_0^{10} dt$$
After integration on putting the given values we get:
$$(10^{-2}kg)\left(\frac{1}{10}-\frac{1}{5}\right)=(-k)(10)$$
(1/5)-(1/10)=(100k)(10)
From above equation the value of k comes out to be:
$k=10^{-4}kgm^{-1}$.

4. Ans. A.
F=6t
Also F=ma, therefore we can write ma=6t
mdv/dt=6t
mdv=6tdt
By integrating both sides of above equation we get:
$$m\int_0^{v} dv=6\int_0^{1} tdt$$
m=1kg
v=3m/s
from the work energy theorem:
$W=\Delta K=1/2m(v^2-u^2)=1/2(1)(9-0)$
=4.5J

5. Ans. A.
The moment of Inertia I of a cylinder is represented by
$I=ml^2/12+mR^2/4$
Or
$I=\frac{m}{4}(l^2/3+R^2)$ (i)
Also, mass=density × volume
Let density be represented by d
Volume of a cylinder is given by=$\pi R^2 l$
$m=\pi R^2 ld$
or $R^2=m/\pi ld$, put this value in equation (i) we get
$I=\frac{m}{4}(l^2/3+m/\pi ld)$
for maxima and minima
$dI/dl=\frac{m}{4}\frac{d}{dl}(l^2/3+m/\pi ld))=0$
$\frac{m}{4}(2l/3-m/\pi l^2d)=0$
$2l/3=m/\pi l^2d$
$2l/3=\pi R^2ld/\pi l^2d$
$2/3=R^2/l^2$
Or
$R^2/l^2=2/3$
$R/l=\sqrt{\left(\frac{2}{3}\right)}$ or $l/R=\sqrt{\left(\frac{3}{2}\right)}$

6. Ans. A.


Let the torque produced be represented by τ
Since it is given that there is negligible friction at the pivot, then the angular acceleration produced can be written as α
Let I be the moment of inertia, then the torque is given by:
$\tau=I\alpha$ (i)
The moment of inertia of a uniform rod of mass M is $Ml^2/3$ (ii)
And torque $\tau=Mg\sin\theta\, l/2$ (iii)
Putting (ii) and (iii) in equation (i) we get:-

$Mg\sin\theta \times l/2 = Ml^2/3 \times \alpha$

Therefore, $\alpha = 3g\sin\theta/2l$.

7. Ans. D.
The acceleration due to gravity inside the earth is : $g=GM/R^3$,which can be represented by a straight line mentioned in the graph of option (D). And the acceleration due to gravity outside the earth: $g=GM/R^2$,which is represented by the exponential decreasing curve in the graph. Therefore, option (D) is the correct answer.

8. Ans. B.
Specific heat,s=heat(q)/mass(m) × change in temperature ΔT,
Heat released = Heat absorbed
Mass of the copper ball=100gm
Mass of water=170gm
The temperature of the system is 75°C and the room temperature is 30°C. The change in temperature of the system when the ball is dropped is given by=75°C-30°C=45°C.
Heat released = Heat absorbed
$m \times s \times (T - 45) = 100 \times 0.1 \times 45 + 170 \times 1 \times 45$ (i)
specific heat of water is 1 cal/gm^{0}C.
By solving equation (i) , T = 885°C.

9. Ans. A.
The thermal expansion,i.e the ratio of the change in volume to original volume is given by: $\frac{\Delta V}{V} = 3\alpha\Delta T$ and when the cube is compressed by the external pressure, then: $\frac{\Delta V}{V} = \frac{P}{K}$. On equating these two equation we get:

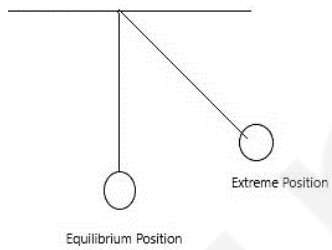
$$3\alpha\Delta T = \frac{P}{K}$$

$$\Delta T = \frac{P}{3\alpha K}$$

10. Ans. C.
$C_p$is the specific heat at constant pressure and C_v is the specific heat at constant volume and R is the universal gas constant, the relation between these quantities is given as following:
C_p-C_v=R, as given in the question a=R/2 for hydrogen gas and b=R/28 for the nitrogen gas, then
$\frac{a}{b} = \frac{28}{2}$, a=14b.

11. Ans. D.
According to the ideal gas equation we know that:
PV=nRT, where n is the no. of moles
Initially let the no. of moles be n_i and the temperature is T_1 which is T_1=17+273=290K
Before heating:
$PV=n_iRT_1=n_iR \times 290$ (i)
After heating let the no. of moles be n_f and the temperature is T_2 which is , T_2= 273+27=300K
After heating:
$PV=n_fR \times 300$ (ii)
From equation (i) and (ii) we have , n_f-n_i= $\frac{PV}{R\times300} - \frac{PV}{R\times290}$
Volume , V = 30m^3 and the Pressure is 10^{5}Pa, by putting the values,
The difference n_f-n_i becomes -2.5 × 10^{25}.

12. Ans. D.



The time taken to reach the equilibrium position is T/4. The kinetic energy is maximum at the equilibrium and is zero at the extreme position. The velocity is given by $V=A\omega\cos wt$ and the kinetic energy, $K=1mv^2/2$. The energy time graph will look like (D).

13. Ans. C.
According to the Doppler's effect in light :
Frequency heard by observer,

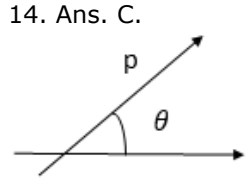
$$\vartheta = \sqrt{\frac{1+\frac{V}{C}}{1-\frac{V}{C}}}(frequency\ of\ source),$$

where C is the speed of light , it is given that observer is moving at half the speed of light then, V= 1/2 C . By inserting this value in above equation we get:

$$\vartheta = \sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}}(10GHz)$$

=17.3 GHz.

14. Ans. C.



In the above figure p is the dipole moment which is a vector quantity and E is the electric field, the torque produced is :

$$\tau = p \times E$$

It is given that when the dipole is subjected to an electric field $E_1=E\hat{i}$, it experiences a torque $T_1=\tau\hat{k}$ and when subjected to another electric field $E_2=\sqrt{3}E_1\hat{j}$ it experiences a torque $T_2=(-T_1)$ then:

$$\tau\hat{k} - \tau\hat{k} = (p_x\hat{i} + p_y\hat{j}) \times (E\hat{i} + \sqrt{3}E\hat{j})$$

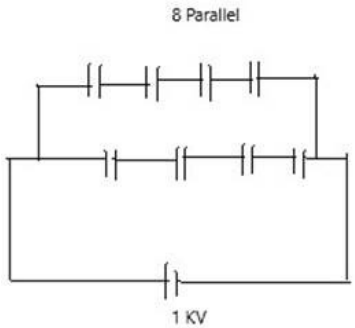
$0= p_xE+p_y\sqrt{3}E=E(p_x+\sqrt{3}p_y)$ and when on considering the $\hat{k}$ direction the equation becomes ,

$0=E^{\hat{k}}(\sqrt{3}p_x - p_y)$

$P_y/p_x = \sqrt{3}$

By taking the slope we have $\theta = 60_o$.

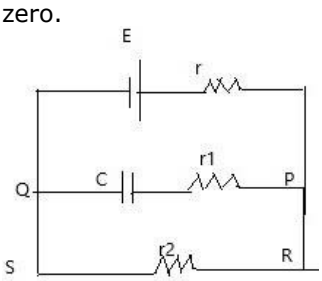
15. Ans. D.



In order to compensate 1KV potential difference minimum, four capacitors are required in series , i.e. C=1/4 for one series .

So, for C_{net} to be 2 μF, 8 Parallel combinations are required .
Minimum no. of capacitors will be = 8 × 4 = 32.

16. Ans. C.
In the steady state condition, the current through PQ is zero.



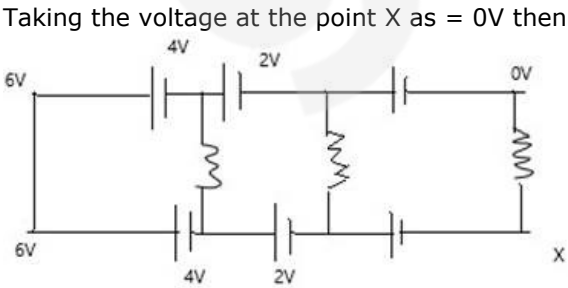
$V_{PQ} = V_{RS}$

$V_{PQ} = (\frac{\epsilon}{r+r_2} \times r_2) - V_{RS}$

Also, $q_c = CV_{PQ}$

$= CE\,(\frac{r_2}{r+r_2})$

17. Ans. D.
Taking the voltage at the point X as = 0V then



Voltage at the other points can be written as shown in the above figure.
Hence, voltage across all resistances is Zero. Hence current = 0.

18. Ans. A.
The time taken by the magnetic needle to complete the oscillation is:

T = $2\pi\sqrt{\frac{I}{MB}}$, where I is the moment of inertia
M is the magnetic moment and B is the magnetic field in Tesla.On the putting the values of I, M and B in the above equation we get:
T = 0.665 sec
For 10 oscillations, the time taken will be = 10 × 0.665 sec = 6.65 sec.

19. Ans. A.
Voltmeter range is 0-10V, then V=IR
$10 = (5 \times 10^{-3}A) \times (15 + r)$
From here r = 1985 ohm or 1.985×10^3ohm.

20. Ans. C.
The resistance of the coil is R=100 ohm (i)

Let the magnetic flux be given by $\emptyset$

And the change in flux be represented by $\Delta\emptyset$

Charge q = $\frac{\Delta\emptyset}{R}$ (ii)

Also, current I = $\frac{dq}{dt}$

So, q = $\int Idt = area\ of\ current\ time\ graph$ = 1/2 $\times 10 \times 0.5\ Coulomb$

= 2.5 Coulomb (iii)
By putting the values from (i) and (iii) equation (ii) we get:

2.5 = $\frac{\Delta\emptyset}{100}$ we get $\Delta\emptyset = 2.5 \times 100 = 250\ Wb.$

21. Ans. A.
Since, it is known that x-rays are produced by electrons liberated from the heated filament and accelerated by a high voltage towards the metal target, the electrons collide with the atoms and nuclei of the metal target as a result of which x-rays are produced. According to the following relation:

eV = $\frac{hc}{\lambda_{min}}$

$$\frac{1}{\lambda_{min}} = \frac{eV}{hc}$$

Taking natural log both sides of above equation :

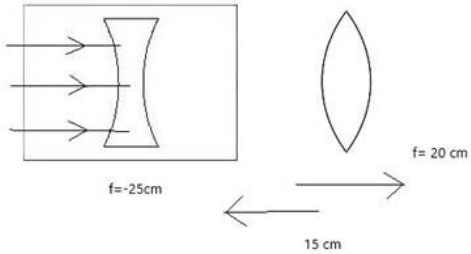
$\ln(\frac{1}{\lambda_{min}}) = lnV + ln\frac{e}{hc}$

$-\ln(\lambda_{min}) = lnV + ln\frac{e}{hc}$

$\ln(\lambda_{min}) = -lnV - ln\frac{e}{hc}$

The above equation is satisfied by the graph given in option (A).

22. Ans. A.
The converging lens is the convex lens while the diverging lens is concave lens.The figure below shows a diverging and converging lens respectively.



The parallel beams are incident on the concave lens thus it forms virtual image at v_1= -25cm , which further works as an object for the convex lens such that f = 20 cm
According to the lens equation:
$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, here for the convex lens i.e converging lens u = -40 cm and f=20 cm . Therefore the final image v will be:

$$\frac{1}{v}-\frac{1}{-40}=\frac{1}{20}$$

By solving above equation v = 40 cm from the converging lens and also the image formed will be real, hence option (A) is correct.

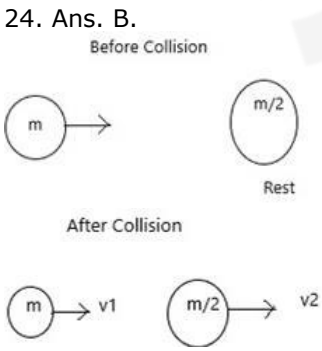
23. Ans. B.
Given that separation between the slits is , d = 0.5 mm = 0.5 × 10^{-3} m
The screen distance from the slits is , D = 150 cm = 1.50m
Let the two wavelengths be $\lambda_1 = 650nm \text{ and } \lambda_2 = 520nm$
For the common maxima:
$n_1\lambda_1 = n_2\lambda_2$
And $n_1650=n_2520$

$$\frac{n_1}{n_2}=\frac{4}{5}$$

Also, $\frac{yd}{D}=n\lambda$

From above relation y = $\frac{Dn\lambda}{d}$=7.8mm.

24. Ans. B.



By the theorem of conservation of linear momentum we can say that the sum of momenta of particle before collision is equal to the sum of momenta of the particles after collision:
From above theorem we can write:
$m_1u_1+m_2u_2=m_1v_1+m_1v_1+m_2v_2$
By putting the values given in the question in above relation we obtain:
$mv=mv_1 + m/2\ v_2$
$2v=2v_1+v_2$ (i)
We can write:
$V_1=v/3$; $v_2=4v/3$
Also, $\lambda_1=\frac{h}{p_1} \text{ and } \lambda_2=\frac{h}{p_2}$
Also, p=mv
So $p_1=mv_1=mv/3$ and $p_2=mv_2=m4v/6$(here m=m/2)
$\frac{\lambda_1}{\lambda_2}$=2/1.

25. Ans. D.
The change in energy is given by:

$$\Delta E=\frac{hc}{\lambda}$$

For λ_1
-E-(-2E)=$\frac{hc}{\lambda_1}$ or $\lambda_1=\frac{hc}{E}$ (i)
Similarly for λ_2
-E-(4E/3)=$\frac{hc}{\lambda_2}$ or $\lambda_2=\frac{3hc}{E}$ (ii)
From (i) and (ii)
The ratio
$\frac{\lambda_1}{\lambda_2}=r=\frac{1}{3}$

26. Ans. B.
The ratio of radioactive nucleus A and the nucleus B is given by:
$\frac{N_B}{N_A}=0.3=N_B=0.3N_A$ (i)
Let the initial no. of nuclei before decay be:
N_o
Now, $N_A+N_B=N_o$
$N_A+0.3N_A=N_o$ (from (i))
$1.3N_A=N_o$
So, $N_A=N_o/1.3$ (ii)
Also we know:
$N_A=N_oe^{-\lambda t}$
$N_o/1.3=N_oe^{-\lambda t}$ from (ii)
$1/1.3=e^{-\lambda t}=\ln(1.3)=\lambda t$
Or t=ln (1.3)/λ
Then t = $\frac{\ln(1.3)}{\frac{\ln(2)}{T}}=\frac{\ln(1.3)T}{\ln(2)}$.

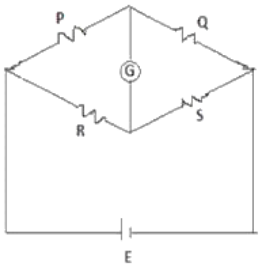
27. Ans. D.
In common emitter amplifier circuit input and output voltages are out of phase. If the input voltage increases then the base current i_b increases , the collector current i_c also increases , so the voltage drop across the resistor

at the collector end that R_c increases. But, the increase in voltage across R_c is in opposite direction. Hence, the difference between the input and output voltage is 180°.

28. Ans. A.
In the amplitude modulation the amplitude of the carrier wave varies in proportion to the waveform being transmitted. Given that the frequency of the carrier wave is represented
as ω_c *and the frequency of the signal to be transmitted is represented as* ω_m
The modulated wave contains the following frequency:
$\omega_c + \omega_m$ *(Upperside band)*, $\omega_c - \omega_m$ *(lowerside band) and* ω_c

So, ω_m will not be contained in the modulated wave.

29. Ans. B.
Consider figure 1



Wheatstone Bridge

The balance condition is :
$\frac{R_1}{R_3} = \frac{R_2}{R_4}$ from above figure the same relation can be written as $\frac{P}{R} = \frac{Q}{S}$ (i)
And if the cell and galvanometer are exchanged , the null point will remain undisturbed and the balanced condition will be :
$\frac{P}{R} = \frac{Q}{S}$, we can see there is no change in the balanced condition, therefore statement (B) is false.

30. Ans. B.
The surface tension, T = $\frac{rhg}{2} \times 10^3\ N/m$
The possible error in the surface tension is given by:
$\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h}$
For calculating the percentage
$100(\frac{\Delta T}{T}) = \left(\frac{10^{-2}\times.01}{1.25\times10^{-2}} + \frac{10^{-2}\times.01}{1.45\times10^{-2}}\right)100$
= 0.01489 × 100%
= 1.489%
= 1.5%

31. Ans. A.
1) C(graphite) + O_2 (g) → CO_2 (g) ; ΔrH °= -393.5 kJ mol^{-1}
2) H_2 (g) + 1/2 O_2 (g) → H_2O(l); ΔrH° = -285.8 kJ mol^{-1}
3) CO_2 (g) +2H_2O(l) →CH_4 (g) +2O_2 (g) ; ΔrH ° = +890.3 kJ mol^{-1}
4) multiplying second reaction by 2 and adding the 3 reactions we will get
C(graphite) +2H_2 (g) → CH_4 (g)
ΔrH will be -74.8 kJ mol^{-1}

32. Ans. D.
Chemical reaction for it:
M_2CO_3 (1gm) + 2HCl→ 2MCl + H_2O + CO_2 (0.01186mole)
From the reaction, we can say that 1 mole of M_2CO_3 will produce I mole of CO_2
So, 1/molar mass(M) =0.01186
M=84.3 gm/mol

33. Ans. A.
From 1st law, change in internal energy=heat supplied+ work done
ΔU = q + w
For adiabatic process : heat exchange q = 0
∴ ΔU = w
∴ Work involve in adiabatic process is at the expense of change in internal energy of the system

34. Ans. D.
b) and d) are the two necessary conditions for observing Tyndall effect.

35. Ans. B.
In FCC unit cell atoms are in contact along face diagonal
So, diagonal on a face= $\sqrt{2}a$ = 4R (2*diameter*2 atoms)
closest distance (2R) = $\sqrt{2}a$ / 2 = $\frac{a}{\sqrt{2}}$

36. Ans. C.
Reducing property increases as electrode potential value decreases.

37. Ans. B.
In benzene, acetic acid associates to form a dimer,so the reaction for it is
$2CH_3COOH \rightarrow (CH_3COOH)_2$
For association,
i=1+(1/n-1)α
αis degree of dissociation
And n is the number of units which associates
For acetic acid,since it dimerises n=2
i=1+(1/2-1)α
i=1-α/2
$\Delta T_f = iK_f m$
Putting the value of i in it,
0.45=(1-α/2)*5.12*(0.2/60)/(20/1000)
1-α/2=0.527
α=0.945

38. Ans. B.
Radius of nth Bohr orbit is given by =0.53n^2 A°
Radius of second Bohr orbit=0.53*2^2 =2.12 A°

39. Ans. B.
We know that the Arrhenius equation is
$K=Ae^{-Ea/RT}$
$K_1 = Ae^{-Ea1/RT}$ .....1)
$K_2 = Ae^{-Ea2/RT}$ .....2)
Dividing 2 by 1 and taking ln, we get $\ln(K_2/K_1)=4$

40. Ans. D.
As the given salt is of weak acid and weak base, so
$pH=7+1/2pK_a-1/2pK_b$
$=7+1/2(3.2)-1/2(3.4)=6.9$

41. Ans. C.
Mg can form basic carbonate like,
$5\ Mg^{2+} + 6C_3^{2-} + 7H_2O \rightarrow 4MgCO_3.Mg(OH)_2.5H_2O + 2HCO_3^-$
Li forms carbonate which is not basic

42. Ans. D.
$O_2 \Rightarrow$ has two unpaired electrons present in π* molecular orbitals
$B_2 \Rightarrow$ Has two unpaired electrons present in π molecular orbitals.
NO⇒One unpaired electron present in π*
Molecular orbital
CO ⇒ has no unpaired electron(Dimagnetic)

43. Ans. C.
In the reaction $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$ oxidation number of xenon is increasing from 4 to 6 while that oxygen is decreasing from +2 to 0,so Xe oxidizes while oxygen reduces, hence it is redox reaction.

44. Ans. A.
NO_3^- : The maximum limit of nitrate in drinking water is 50 ppm. Excess nitrate in drinking water can cause disease such as methemoglobinemia.
SO_4^{2-} : above 500 ppm of SO_4^{2-} ion, in drinking water it causes laxative effect otherwise at moderate levels it is harmless
F^- : Above 2ppm concentration of F^- in drinking water cause brown mottling of teeth.
∴ The concentration given in question of SO_4^{2-}-& NO_3^- in water is suitable for drinking but the concentration of F^- (i.e 10 ppm) make water unsuitable for drinking purpose.

45. Ans. C.
All the species are isoelectronic, they contain same number of electrons(10)

46. Ans. A.
When chlorine gas reacts with cold and dilute aqueous NaOH,
$Cl_2 + 2OH^-$ (COLD AND DILUTE) $\rightarrow Cl^- + ClO^- + H_2O$

47. Ans. B.
(a) ZnO+Na2O → Na2ZnO2 here ZnO acts as an acid
(b) ZnO+CO2 → ZnCO3 here ZnO acts as an base ZnO is an amphoteric oxide but in given reaction, it acts as acid and base in a) and b) reactions.

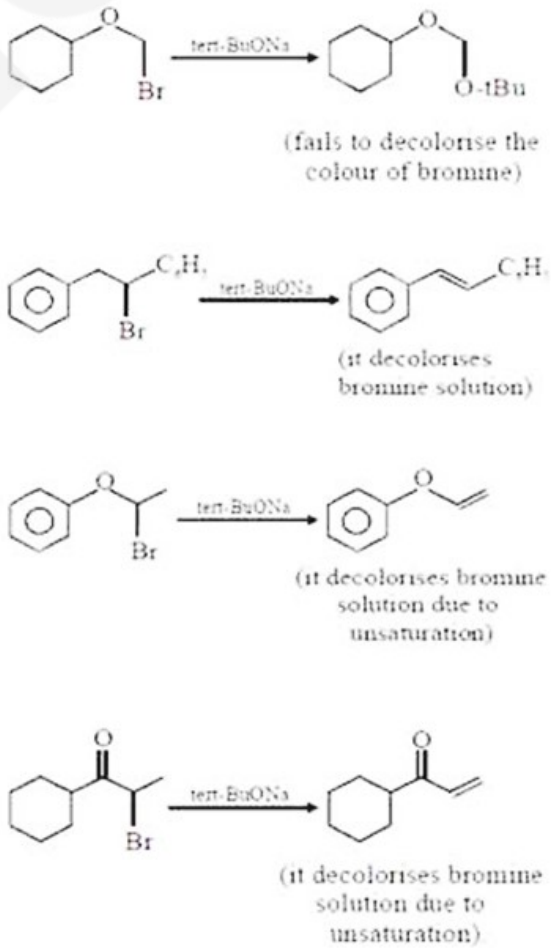
48. Ans. B.
Sodium salt of oxalate will produce effervescence with conc.H_2SO_4
$Na_2C_2O_4 + H_2SO_4 \rightarrow Na_2SO_4 + CO\uparrow + CO_2\uparrow + H_2O$
$Na_2C_2O_4 + CaCl_2 \rightarrow Ca(C_2O_4)$ [white ppt] + 2NaCl
$5\ Ca(C_2O_4) + 2KMnO_4$(purple) $+8\ H_2SO_4 \rightarrow K_2SO_4 + 5CaSO_4 + 2MnSO_4$ (Colourless) $+10CO_2 + 8H_2O$

49. Ans. A.
Mass in the body of a healthy human adult has: Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%).
Since H is 10% and ^1H atoms are replaced by ^2H atoms then weight of hydrogen is just doubled. Weight gain will be 75 x .1 = 7.5 kg

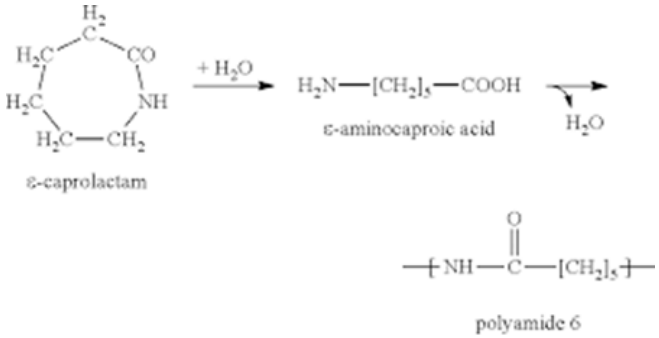
50. Ans. B.
Molarity =moles of complex/volume(in l)
0.1M=moles*1000/100
Moles of complex=0.01 mole
Moles of ions precipated= no of ions/N_A=$1.2*10^{22}/6.02*10^{23}$=0.02 moles
Number of Cl^- present in ionization sphere=moles of ions precipated/moles of complex=2
This shows that only 2 Cl^- are present in the ionization sphere.

51. Ans. A.
i) we know that the reagent used in the process of nitration is conc. HNO_3 and conc. H_2SO_4 . So, aniline will act as a base and its protonation takes place in presence of H_2SO_4
ii) anilium ion formed is strongly deactivating and meta directing so it gives meta nitration product as major product.

52. Ans. C.



53. Ans. C.
Nylon-6 is formed by hydolysing its monomer unit, caprolactum

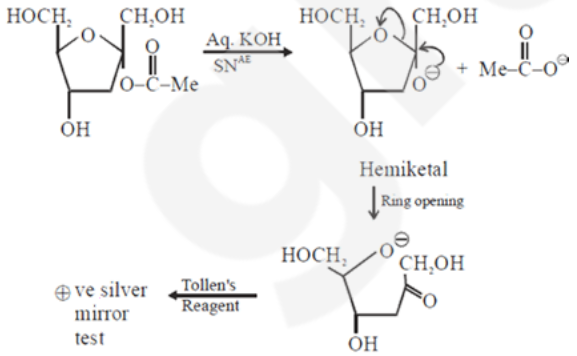


54. Ans. B.
B is non aromatic since it has no conjugation so it is least stable while other 3 are aromatic in nature

55. Ans. D.
For any SN1 reaction reactivity is decided by ease of dissociation of alkyl halide. Higher the stability of R^+ (carbocation) higher would be reactivity of SN1 reaction.
$R\text{-}X \rightarrow R^+ + X^-$
Stability of carbocation
$CH_3\text{-}CH_2\text{-}CH_2^+ < CH_3\text{-}CH^+\text{-}CH_2\text{-}CH_3 < P\text{-}H_3CO\text{-}C_6H_4\text{-}CH_2^+$
Since stability of cation follows order. Hence correct order is2<1<3.

56. Ans. D.
Elimination reaction is highly favoured by
(a) Bulkier base
(b) High temperature
The above reaction is biomolecular elimination reaction giving $C_6H_5CH = CHC_6H_5$ as major product.

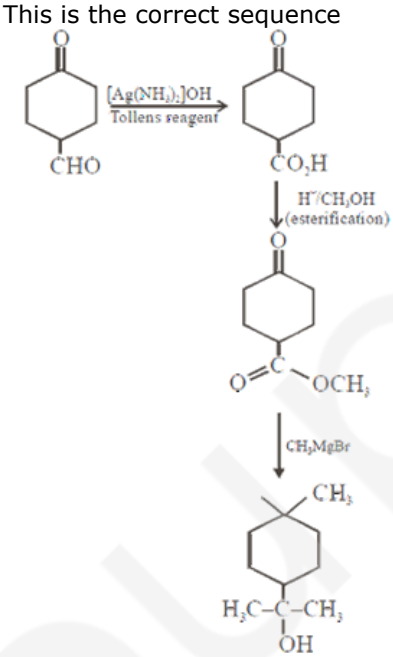
57. Ans. B.
1) In presence of Aqueous KOH solution,esters give SNAE reaction so following reaction takes place



(2) In above compound in presence of Aq. KOH, (SNAE) reaction takes place & ∝-Hydroxy carbonyl compound is formed which give +ve Tollen's test .So, this compound behave as reducing sugar in an aqueous KOH solution.

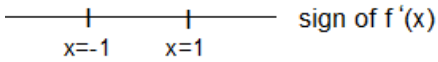
58. Ans. B.
Anti markownikov product since addition of HBr in presence of peroxide is anti markownikov type of reaction (4 stereo isomers possible because of 2 chiral centre. The molecule is nonsymmetric)

59. Ans. C.
This is the correct sequence

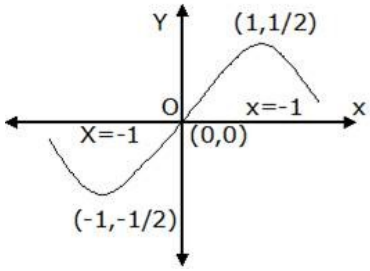


60. Ans. D.
DIBAL-H is electrophilic reducing agent, it will reduce esters,carboxylic acids, cyanides,amides into aldehydes by partial reduction.

61. Ans. B.
$f : R \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$
$f(x) = \frac{x}{1+x^2} \ \forall \ x \in R$
Or f `(x) = $\frac{(1+x^2).1-x.2x}{(1+x^2)^2} = -\frac{(x+1)(x-1)}{(1+x^2)^2}$
For f(x) = -1/2, x = -1 and f(x) = 1/2, x = 1
Sign of f `(x) changes as



Hence graph of f(x) will look like



From above graph we can see that x and y have a many to one relationship.
Hence f(x) is a surjective function.

62. Ans. C.
The quadratic equation can be written as
$\sum_{r=1}^{n}(x+r-1)(x+r) = 10n$
$\rightarrow \sum_{r=1}^{n}\left(x^2 + (2r-1)x + (r^2 - r)\right) = 10n$
On solving we get

$$x^2 + nx + \left(\frac{n^2-31}{3}\right) = 0$$

Let α and (α+1) be the roots of this quadratic equation
Put x=α in the equation

$$2\alpha + 1 = -n \Rightarrow \alpha = -\frac{(n+1)}{2} \quad (1)$$

Put x=(α+1)

$$\alpha(\alpha+1) = \frac{(n^2-31)}{3} \quad (2)$$

Using equation (1) and (2)

$$n^2 = 121$$

Or n = 11

63. Ans. D.

$$z = i\sqrt{3} \Rightarrow 2\omega + 1 = i\sqrt{3}$$

$$\Rightarrow \omega = \frac{-1+i\sqrt{3}}{2}$$

$$\omega^2 = -\frac{(1+i\sqrt{3})}{2}, \omega^3 = 1$$

$$\Rightarrow 1 + \omega + \omega^2 = 0, \omega^7 = \omega^6 \times \omega = \omega$$

$R_1 \rightarrow R_1+R_2+R_3$

$$\begin{vmatrix} 3 & 0 & 0 \\ 1 & -\omega^2 - 1 & \omega^2 \\ & 1 & \omega^2 \ \omega \end{vmatrix}$$

$$= 3(-1 - \omega - \omega)$$

= -3z
Hence k = -z

64. Ans. A.

Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$,

$$3A^2 = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$$

$$12A = \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$$

$$\therefore 3A^2 + 12A = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$adj(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

65. Ans. C.

Determinant $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix}$

$R_1 \rightarrow R_1 - R_2$

$$D = \begin{vmatrix} 0 & 1-a & 0 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix}$$

$$D = -(1-a)(1-a) = -(1-a)^2 = 0$$

$$a = 1$$

For $a = 1$ we have first two planes coincident.
Now two different planes are

$$x + y + z = 1$$

$$x + by + z = 0$$

For no solutions these two lines should be parallel

$$\frac{1}{1} = \frac{1}{b}$$

Hence b=1
Hence b={1} is a singleton set

66. Ans. D.
X – 4 ladies, 3 men
Y – 3 ladies, 4 men
Ways of selection

X	Y	+	X	Y	+	X	Y	+	X	Y
0L	3L		1L	2L		2L	1L		3L	0L
3M	0M		2M	1M		1M	2M		0M	3M

Number of ways of selection

$= {}^4C_0.{}^3C_3.{}^3C_3.{}^4C_0 + {}^4C_1.{}^3C_2.{}^3C_2.{}^4C_1 + {}^4C_2.{}^3C_1.{}^3C_1.{}^4C_2 + {}^4C_3.{}^3C_0.{}^3C_0.{}^4C_3$
= 485

67. Ans. C.
$({}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + \ldots {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 \ldots {}^{10}C_{10}) = S_1 - S_2$
$S_1 = (1/2)({}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + \ldots {}^{21}C_{20})$
$= (1/2)({}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + \ldots {}^{21}C_{21} - 2)$
$= 2^{20} - 2$
$S_2 = ({}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 \ldots {}^{10}C_{10}) = 2^{10} - 1$
Hence $S_1 - S_2 = 2^{20} - 2^{10}$

68. Ans. D.

$$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$$

$$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - (15a)(5c) - (15a)(3b) - (3b)(5c) = 0$$

$$\Rightarrow \frac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0$$

It is possible when 15a = 3b = 5c

$$\therefore b = \frac{5c}{3}, a = \frac{c}{3}$$

$$\Rightarrow a + b = 2c$$

Hence b, c, a are in AP

69. Ans. D.

$f(x) = ax^2 + bx + c$

$f(1) = a + b + c = 3$

Now $f(x+y) = f(x) + f(y) + xy$

Put y=1

$f(x+1) = f(x) + f(1) + x$

$f(x+1) = f(x) + x + 3$

Now $f(2) = 7, f(3) = 12$

Now $Sn = 3 + 7 + 12 \ldots Tn$ (1)

Also $Sn = 3 + 7 + \cdots T(n-1) + Tn$ (2)

Subtracting (2) from (1)

$Tn = 3 + 4 + 5 + \cdots upto\ n\ terms$

$Tn = \frac{(n^2+5n)}{2}$

$Sn = \sum Tn = \sum \frac{(n^2+5n)}{2}$

$Sn = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2}\right]$

$S_{10} = 330$

70. Ans. A.

$\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi-2x)^3} = \lim_{x\to\frac{\pi}{2}} \frac{\cot x(1-\sin x)}{-8\left(x-\frac{\pi}{2}\right)^3}$

$= \lim_{x\to\frac{\pi}{2}} \frac{\tan\left(\frac{\pi}{2}-x\right)}{8\left(\frac{\pi}{2}-x\right)} \frac{\left(1-\cos\left(\frac{\pi}{2}-x\right)\right)}{\left(\frac{\pi}{2}-x\right)^2}$

$= \frac{1}{8}.1.\frac{1}{2} = \frac{1}{16}$

71. Ans. D.

Let

$y = \tan^{-1}\left(\frac{2\times 3x\sqrt{x}}{1-(3x\sqrt{x})^2}\right) =$

$2\tan^{-1}(3x\sqrt{x})$

$\frac{dy}{dx} = 2 \times \frac{1}{1+9x^3} \times 3 \times \frac{3}{2} \times \sqrt{x}$

$\frac{dy}{dx} = \sqrt{x}\left(\frac{9}{1+9x^3}\right)$

Hence $g(x) = \left(\frac{9}{1+9x^3}\right)$

72. Ans. A.

$y = \frac{(x+6)}{(x-2)(x-3)}$

Point of intersection with y axis is (0,1)

$\frac{dy}{dx} = \frac{(x^2-5x+6)(1)-(x+6)(2x-5)}{(x^2-5x+6)^2}$

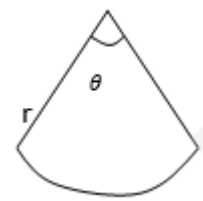
$\frac{dy}{dx} = 1$ at point (0,1)

Slope of normal = -1

Hence equation of normal is $x + y = 1$

$\therefore \left(\frac{1}{2}, \frac{1}{2}\right)$ satisfies the normal.

73. Ans. B.



Total length = $r + r + r\theta = 20$

$\theta = \frac{20-2r}{r}$

Area = $\frac{1}{2}r^2\theta = \frac{1}{2}r^2\left(\frac{20-2r}{r}\right)$

$A = 10r - r^2$

for maximum area $\frac{dA}{dr} = 0$

$\Rightarrow 10 - 2r = 0, r = 5$

$A = 50 - 25 = 25$

74. Ans. A.

$I_4+I_6 = \int (tan^4 x + tan^6 x)\,dx$

$= \int tan^4 x(1 + tan^2 x)\,dx$

$= \int tan^4 x.sec^2 x\,dx = \frac{1}{5}tan^5 x + c$

$a = \frac{1}{5}, b = 0$

75. Ans. A.

$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ (1)

Using property

$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1-\cos x}$ (2)

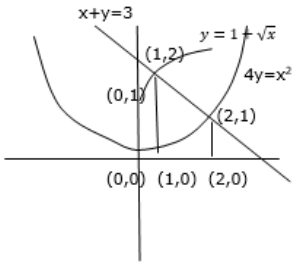
On adding (1) and (2)

$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{2\,dx}{1-cos^2 x}$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} cosec^2 x\, dx$$

$$I = [-cotx] = 2$$

76. Ans. C.



Area $A = \int_0^1 (1+\sqrt{x})\,dx + \int_1^2 (3-x)dx - \int_0^2 \frac{x^2}{4}dx$

$$A = \left[x + \frac{x^{3/2}}{3/2}\right] + \left[3x - \frac{x^2}{2}\right] + \left[\frac{x^3}{12}\right]$$

$$A = \frac{5}{2}$$

77. Ans. D.

$$(2+\sin x)\frac{dy}{dx} + (y+1)\cos x = 0$$

$$\Rightarrow \frac{d}{dx}[(2+\sin x)(y+1)] = 0$$

$$\Rightarrow (2+\sin x)(y+1) = c$$

$$x = 0, y = 0 \Rightarrow c = 4$$

$$y + 1 = \frac{4}{2+\sin x}$$

$$y\left(\frac{\pi}{2}\right) = \frac{4}{3} - 1 = \frac{1}{3}$$

78. Ans. C.

$$\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = \pm 56$$

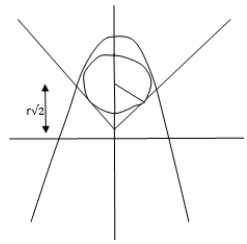
$$\Rightarrow 5k^2 + 13k - 46 = 0$$

Or $5k^2 + 13k + 66 = 0$ (no real solutions)

$$\therefore k = -\frac{23}{2}, or\ k = 2$$

As k is an integer, so k=2
Vertices are A(2,-6), B(5,2), C(-2,2)
Equation of altitude dropped from vertex A is x=2 ….(1)
Equation of altitude dropped from vertex C is 3x+8y-10=0 ….(2)
Solving (1) and (2)
Orthocenter is (2, 1/2)

79. Ans. B.



Due to symmetry the center of circle must lie on y axis
Let center of circle be (0,k)
Length of perpendicular from (o,k) toy=x

$$r = \left|\frac{k}{\sqrt{2}}\right|$$

∴equation of circle : $x^2 + (y-k)^2 = \frac{k^2}{2}$

Solving circle and parabola

$$4 - y + y^2 - 2ky + \frac{k^2}{2} = 0$$

$$\Rightarrow y^2 - (2k+1)y + \left(\frac{k^2}{2} + 4\right) = 0$$

Because circle touches the parabola
D=0

$$(2k+1)^2 = 4\left(\frac{k^2}{2} + 4\right)$$

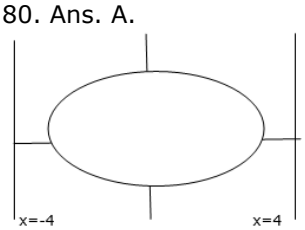
$$\Rightarrow 4k^2 + 4k + 1 = 2k^2 + 16$$

On solving we get

$$k = \frac{=4+\sqrt{136}}{4}$$

Thus radius $r = \frac{k}{\sqrt{2}} = 4(\sqrt{2}-1)$

80. Ans. A.



eccentricity of ellipse = 1/2

Now, $-\frac{a}{e} = -4 \Rightarrow a = 2$

$$\therefore b^2 = a^2(1-e^2) = 3$$

Equation of ellipse

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$\Rightarrow \frac{x}{2} + \frac{2y}{3} \times y' = 0 \Rightarrow y' = -\frac{3x}{4y}$$

$$y'\left(1, \frac{3}{2}\right) = -\frac{1}{2}$$

$$\therefore equation\ of\ normal\ at\ \left(1, \frac{3}{2}\right)$$

$$\left(y - \frac{3}{2}\right) = 2(x-1)$$

$$4x - 2y = 1$$

81. Ans. A.

Equation of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Foci is $(\pm 2,0)$ hence $ae=2 \Rightarrow a^2e^2=4$

$b^2=a^2(e^2-1)$

$\therefore a^2+b^2=4$ (1)

Hyperbola passes through $(\sqrt{2},\sqrt{3})$

$\therefore \frac{2}{a^2}-\frac{3}{b^2}=1$ (2)

On solving (1) and (2)

$a^2=8\ (b^2\ is\ negative\ so\ rejected)$

$\therefore a^2=1\ and\ b^2=3$

Equation of hyperbola

$\frac{x^2}{1}-\frac{y^2}{3}=1$

$\Rightarrow 2x-\frac{2yy'}{3}=0 \Rightarrow y'=\frac{3x}{y} \Rightarrow$

$y'^{(\sqrt{2},\sqrt{3})}=\sqrt{6}$

Slope of normal = $-\frac{1}{\sqrt{6}}$

Equation of normal at $(\sqrt{2},\sqrt{3})$

$\frac{\sqrt{2}x}{1}-\frac{\sqrt{3}y}{3}=1$

Hence $(2\sqrt{2},3\sqrt{3})$ satisfies this normal.

82. Ans. A.

Normal vector

$\begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix}=5i+7j+3k$

So plane is $5(x-1)+7(y+1)+3(z+1)=0$

$\Rightarrow 5x+7y+3z+5=0$

Distance $\frac{5+(7\times3)+(3\times-7)+5}{\sqrt{5^2+7^2+3^2}}=\frac{10}{\sqrt{83}}$

83. Ans. A.

Line PQ : $\frac{x-1}{1}=\frac{y+1}{4}=\frac{z-3}{5}=p$

Let F$(p+1,4p-2,5p+3)$ be the point of intersection of line PQ and given plane.

F lies on the plane

$2(p+1)+3(4p-2)-4(5p+3)+22=0$

$\Rightarrow -6p+6=0 \Rightarrow p=1$

F(2,2,8)

$PF=\sqrt{(2-1)^2+(2-(-2))^2+(8-3)^2}=\sqrt{42}$

$PQ=2\times PF=2\sqrt{42}$

84. Ans. A.

$a=2i+j-2k, b=i+j\ and\ |a|=3$

$a\times b=2i-2j+k \Rightarrow |a\times b|=3$

Now : $|(a\times b)\times c|=|a\times b||c|\sin 30$

$\Rightarrow |(a\times b)\times c|=3|c|\times\frac{1}{2}$

$\Rightarrow 3=3|c|\times\frac{1}{2}$

$\Rightarrow |c|=2$

Now : $|c-a|=3$

$\Rightarrow c^2+a^2-2c.a=9$

$\Rightarrow 4+9-2a.c=9$

$\Rightarrow a.c=2$

85. Ans. D.

15 green + 10 yellow =25 balls

Probability $P(green)=\frac{15}{25}=\frac{3}{5}=p$

Probability $P(yellow)=\frac{10}{25}=\frac{2}{5}=q$

n=10

variance $=npq=10\times\frac{3}{5}\times\frac{2}{5}=\frac{12}{5}$

86. Ans. A.

P(exactly one of A or B occurs)

$=P(A)+P(B)-2P(A\cap B)=1/4$ (1)

P(exactly one of B or C occurs)

$=P(B)+P(C)-2P(B\cap C)=1/4$ (2)

P(exactly one of C or A occurs)

$=P(C)+P(A)-2P(C\cap A)=1/4$ (3)

Adding (1), (2) and (3)

$2\sum P(A)-2\sum P(A\cap B)=\frac{3}{4}$

$\Rightarrow \sum P(A)-\sum P(A\cap B)=\frac{3}{8}$

Now, $P(A\cap B\cap C)=\frac{1}{16}$ (Given)

$\therefore P(A \cup B \cup C)$
$= \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$
$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$

87. Ans. D.

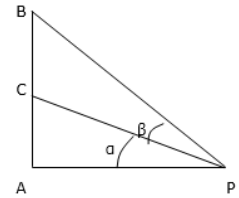
Let $A = \{0,1,2,3 \ldots .10\}$
N(S) = $^{11}C_2$=55
Let E be the given event
$\therefore E = \{(0,4),(0,8),(2,6),(2,10),(4,8),(6,10)\}$
N(E) = 6
$\therefore P(E) = \frac{6}{55}$

88. Ans. C.

$5(tan^2 x - cos^2 x) = 2\cos 2x + 9$
$\Rightarrow 5\left(\frac{1-cos^2 x}{cos^2 x} - cos^2 x\right) = 2(2cos^2 x - 1) + 9$
Let $cos^2 x = t$
$\Rightarrow 5\left(\frac{1-t}{t} -\right) = 2(2t-1) + 9$
$\Rightarrow 5(1 - t - t^2) = t(4t + 7)$
$\Rightarrow 9t^2 + 12t - 5 = 0$
$\Rightarrow (3t-1)(3t+5) = 0$
$\Rightarrow t = \frac{1}{3}\ or\ t = -\frac{5}{3}(not\ possible)$
So $t = \frac{1}{3}$

$\cos 2x = 2\left(\frac{1}{3}\right) - 1 = -\frac{1}{3}$
$\cos 4x = 2\left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$

89. Ans. B.



$\frac{AB}{AP} = \frac{1}{2}$
Let $\angle APC = \alpha$
$\tan\alpha = \frac{AC}{AP} = \frac{1}{2}.\frac{AB}{AP} = \frac{1}{4}$
Now, $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{AB}{AP} = \frac{1}{2}$
On solving $\tan\beta = \frac{2}{9}$

90. Ans. D.

$(p \to q) \to [(\sim p \to q) \to q]$
$(p \to q) \to [(p \vee q) \to q]$
$(p \to q) \to [(\sim p \wedge \sim q) \vee q]$
$(p \to q) \to [(\sim p \vee q) \wedge (\sim q \vee q)]$
$(p \to q) \to (p \to q)$
Hence it is a tautology.

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