1.Consider the following two binary relations on the set A={a, b, c} :
R_1={(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)} and R_2={(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c)}.
Then :

A. both R_1 and R_2 are not symmetric.

B. R_1 is not symmetric but it is transitive.

C. R_2 is symmetric but it is not transitive.

D. both R_1 and R_2 are transitive.

Answer ||| C

Solution ||| We have set as:
Set A={a, b, c} :
R_1={(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)} and
R_2={(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c)}

R_1 would be symmetric over a set A, if it holds for all elements in set A that 'a' or 'b' or 'c' is related to 'b' or 'c' or 'a' if and only if 'b' or 'c' or 'a' is realted to 'a' or 'b' or 'c'.

R_1 would be transitive over a set A, if an element 'a' is related to 'b' and 'b' is related to 'c' then 'a' is related to 'c'.
Now,
(b,c) $\in R_1$ but (c,b) $\notin R_1$
=> Not R_1 is not symmetric

Now
(b,c)$\in R_1$, (c,a) $\in R_1$, but (b,a)$\notin R_1$
=> R_1 is not transitive

(a,b)$\in R_2$ and (b,a) $\in R_2$
Thus, R_2 is symmetric

But
(b,a) $\in R_2$ and (a,c) $\in R_2$
but (b,c) $\notin R_2$
=> R_2 is not transitive.

2.If $\lambda \in R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is :

A. $4\sqrt{2}$

B. $2\sqrt{5}$

C. $2\sqrt{7}$

D. 20

Answer ||| B

Solution ||| $x^2 + (2-\lambda)x + (10-\lambda) = 0$

Let a and b be the 2 roots

$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$= \quad (\lambda-2)^3 - 3(10-\lambda)(\lambda-2)$

$= \quad \lambda^3 - 3\lambda^2 - 24\lambda + 52$

$\frac{dz}{d\lambda} = 3\lambda^2 - 6\lambda - 24 = 0$

$(\lambda-4)(\lambda+2) = 0$

$\frac{d^2z}{d\lambda^2} =$ 6λ-6

$Now\ putting\ the\ value\ of\ \lambda\ we\ get$

$a^3 + b^3 is\ \max at - 2\ and\ \min at\ 4\ as\ \frac{d^2z}{d\lambda^2} < 0 at\ \lambda = -2\ and > 0\ at\ \lambda = 4$

Equation is $x^2 - 2x + 6 = 0$

Hence roots are $1 \pm \sqrt{5}i$

The difference=$2\sqrt{5}i$

And magnitude=$2\sqrt{5}$

3.The set of all α ∊ R, for which $w = \frac{1+(1-8\alpha)z}{1-z}$ is a purely imaginary number, for all z ∊ Csatisfying |z| = 1 and Re z ≠ 1, is :

A. an empty set

B. {0}

C. $\left\{0, \frac{1}{4}, -\frac{1}{4}\right\}$

D. equal to R

Answer ||| B

Solution ||| When a complex number is purely imaginary then summation of the complex number and its conjugate would vanish

=> Z + $\overline{Z}$ = 0

Hence,

$$\frac{1+(1-8\alpha)z}{1-z}+\frac{1+(1-8\alpha)\bar{z}}{1-\bar{z}}=0$$

On cross multiplying

$$(1-\bar{z})+(1-8\alpha)z(1-\bar{z})(1-z)+(1-z)+(1-8\alpha)\bar{z}(1-z)=0$$

$$(2-z-\bar{z})+(1-8\alpha)(z+\bar{z}-2)=0$$

$$(z+\bar{z}-2)(8\alpha)=0$$

$$\alpha=0$$

4.Let A be a matrix such that A. $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ is a scalar matrix and $|3A|=108$. Then A^2 equals :

A. $\begin{bmatrix}4 & -32\\0 & 36\end{bmatrix}$

B. $\begin{bmatrix}36 & 0\\-32 & 4\end{bmatrix}$

C. $\begin{bmatrix}4 & 0\\-32 & 36\end{bmatrix}$

D. $\begin{bmatrix}36 & -32\\0 & 4\end{bmatrix}$

Answer ||| D

Solution |||

Let, $A=\begin{vmatrix}a & b\\c & d\end{vmatrix}$

Now, a scalar matrix is a diagonal matrix having equal value in the elements of the main diagonal.

Now, given that, A. $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ is scalar

=>A. $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ = $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix}$

=>

A. $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ = $\begin{bmatrix}a & 2a+3b\\c & 2c+3d\end{bmatrix}$

Since the above matrix is scalar

=> c=0…….(1)

Also, 2a+3b=0…..(2)

Also, a=2c+3d….(3)

But c = 0

=> $a = 3d$......(4)

Now,

$|3A| = 108.$

=> ad - bc = 12

But a = 3d and c = 0

=> d^2 = 4

=> d = +2, -2

So, a = 3d = +6, -6

b = -2a/3 = +4, -4

Thus, $A^2 = \begin{vmatrix} a^2 & ab+bd \\ c(a+d) & bc+d^2 \end{vmatrix} = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix} or \begin{bmatrix} 36 & 32 \\ 0 & 4 \end{bmatrix}$

5.Let S be the set of all real values of k for which the system of linear equations

$x + y + z = 2$

$2x + y - z = 3$

$3x + 2y + kz = 4$

has a unique solution. Then S is :

A. an empty set

B. equal to {0}

C. equal to R

D. equal to R−{0}

Answer ||| D

Solution ||| Given set of the linear equation is:

$x + y + z = 2$

$2x + y - z = 3$

$3x + 2y + kz = 4$

The above equations have a unique solution.

Thus, |A|$\neq$0

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$AX = B$

=> 1(k + 2) -1(2k + 3) + 1(4 - 3) $\neq$ 0

=> k + 2 - 2k - 3 + 1 $\neq$ 0

=> k$\neq$ 0

=> equal to R−{0}

6.n-digit numbers are formed using only three digits 2, 5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is :

A. 6

B. 7

C. 8

D. 9

Answer ||| B

Solution ||| As n-digit number is to be formed using 2, 5 and 7

These numbers can be used n times,

So total distinct numbers possible=3^n

We need atleast 900 distinct numbers, thus we will use greater than equal to.
Hence,

$$3^n \geq 900$$
$$n = 7$$

7.If n is the degree of the polynomial, $\left[\frac{2}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8 + \left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$ and m is the coefficient of x^n in it, the ordered pair (n, m) is equal to :

A. $(24, (10)^8)$

B. $(8, 5\,(10)^4)$

C. $(12, (20)^4)$

D. $(12, 8(10)^4)$

Answer ||| C

Solution ||| Degree is the largest exponent of the variable.

We will simplify the equation to find out the largest exponent of the variable.
Rationalizing the denominator, we have

$$=\left[\frac{2\left(\sqrt{5x^3+1}+\sqrt{5x^3-1}\right]}{2}\right]^8+\left[\frac{2\left(\sqrt{5x^3+1}-\sqrt{5x^3-1}\right]}{2}\right]^8$$
$$=2\left[C(8,0)\sqrt{5x^3+1}^8+C(8,2)\sqrt{5x^3+1}^6\sqrt{5x^3-1}^2+\cdots\sqrt{5x^3-1}^8\right]$$
$$=2[(5x^3+1)^4+28(5x^3+1)^3(5x^3-1)+\cdots+(5x^3-1)^4]$$

Now, the highest exponent would be that of $(x^3)^4$. We do not need to expand the complete binomial, just we have to find the largest exponents.
Thus,

n = 12,

And the coefficient of x^n would be

$$m = 2(5^4 + 140.5^2 + 70.5^4 + 140.5^2 + 5^4)$$
$$= 160000 = 20^4$$

Thus, n = 12 and m = 20^4

8.If $x_1, x_2, \ldots, x_n$ and $\frac{1}{h_1}, \frac{1}{h_2}, \ldots, \frac{1}{h_n}$ are two A.P.s such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 \cdot h_{10}$ equals :

A. 2560

B. 2650

C. 3200

D. 1600

Answer ||| A

Solution ||| Given that,

$x_1, x_2, \ldots, x_n$ are in AP.

Also, x_3= 8 and x_8 = 20

Since, x_3= 8

=> x_1+ 2d = 8 -----(1)

And, x_8 = 20

=> x_1 + 7d = 20 -----(2)

From above two equations, (2) - (1)

=> 5d = 12

=>d=12/5

and

a = 8 - 2 (12/5) = 16/5

Thus, x_5 = x_1 + 4d = 16/5 + 48/5

=> x_5 = 64/5

Also, given that $\frac{1}{h_1}, \frac{1}{h_2}, \ldots, \frac{1}{h_n}$ are in AP

$$\frac{1}{h_2}=\frac{1}{h_1}+D$$

....(3)

$$\frac{1}{8}=\frac{1}{h_1}+D$$

and

$$\frac{1}{h_7}=\frac{1}{h_1}+6D$$

....(4)

$$\frac{1}{20}=\frac{1}{h_1}+6D$$

Substract equation (3) from (4).

$$5D=\frac{1}{20}-\frac{1}{8}$$

$$5D=\frac{-3}{40}$$

$$D=\frac{-3}{200}$$

So,

$$\frac{1}{h_1}=\frac{1}{8}-D$$

$$\frac{1}{h_1}=\frac{1}{8}-\frac{-3}{200}$$

$$=\frac{28}{200}=\frac{7}{50}$$

$$\frac{1}{h_{10}}=\frac{1}{h_1}+9D$$

$$=\frac{7}{50}-\frac{3\times 9}{200}$$

$$=\frac{28-27}{200}=\frac{1}{200}$$

$$h_{10}=200$$

So,

$$x_5h_{10}=\frac{64}{5}\times 200$$

$$x_5h_{10}=2560$$

9.If b is the first term of an infinite G.P. whose sum is 5, then b lies in the interval :

A. (−∞, −10]

B. (−10, 0)

C. (0, 10)

D. [10, ∞)

Answer ||| C

Solution ||| Given that the first term is b and the sum of an infinite GP is 5 .

Sum of an infinite GP = a/1-r, where a is the first term and |r|<1

=> $5 = \frac{b}{1-r}$

=> $1 - r = \frac{b}{5}$

=>$r = 1 - \frac{b}{5}$

=>$r = \frac{5-b}{5}$

As |r|<1

=> $-1 < r < 1$

=>$-1 < \frac{5-b}{5} < 1$

=>$-5 < 5 - b < 5$

=>$-10 < -b < 0$

=>$0 < b < 10$

Thus, $b \in (0,10)$

10.If $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2 \sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$, then $\lim_{x \to 0} \frac{f'(x)}{x}$

A. does not exist.

B. exists and is equal to 2.

C. exists and is equal to 0.

D. exists and is equal to −2.

Answer ||| D

Solution ||| $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2 \sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

To calculate f'(x) from f(x) when f(x) is in determinant form, we differentiate each column/row separately.

Thus,

$$f'(x) = \begin{vmatrix} -\sin x & 1 & 0 \\ 2 \sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ 2 \cos x & 2x & 2 \\ \tan x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ 2 \sin x & x^2 & 2x \\ sec^2 x & 1 & 0 \end{vmatrix}$$

But, $\begin{vmatrix} \cos x & x & 1 \\ 2\cos x & 2x & 2 \\ \tan x & x & 1 \end{vmatrix} = 0$,because first and second row is proportional

$$\lim_{x\to 0}\frac{f'(x)}{x} = \frac{1}{x}.\begin{vmatrix} -\sin x & 1 & 0 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix} + \frac{1}{x}.\begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ sec^2 x & 1 & 0 \end{vmatrix}$$

Now in matrix, we will multiply 1/x to any one of the row or column. We will select the one which can be solved easily(the limits)

Thus,

$$\lim_{x\to 0}\frac{f'(x)}{x} = \begin{vmatrix} -\frac{\sin x}{x} & 1 & 0 \\ \frac{2\sin x}{x} & x^2 & 2x \\ \frac{\tan x}{x} & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ \frac{2\sin x}{x} & \frac{x^2}{x} & \frac{2x}{x} \\ sec^2 x & 1 & 0 \end{vmatrix}$$

$$\lim_{x\to 0}\frac{f'(x)}{x} = \begin{vmatrix} -1 & 1 & 0 \\ 2 & 0 & 0 \\ 1 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 1 & 1 & 0 \end{vmatrix}$$

=$-2(1) - 1(2-2)$

=-2

11.If $x^2 + y^2 + \sin\ y = 4$, then the value of $\frac{d^2y}{dx^2}$ at the point (-2, 0) is :

A. -34

B. -32

C. 4

D. -2

Answer ||| A

Solution ||| $Let\ z = x^2 + y^2 + \sin y = 4$

Differentiation the above equation using chain rule of differentiation

=>2x + 2y y' + cos y y' = 0 , where y' = dy/dx

=> y' = $\frac{-2x}{2y\ +\ \cos y}$

$$\frac{dy}{dx} = -\frac{2x}{2y + \cos y} = 4\ at\ (-2{,}0)$$

$$\frac{d^2y}{dx^2} = \frac{-2(2y + \cos y) + \frac{2x(2 - \sin y)dy}{dx}}{(2y + \cos y)^2}$$

$$\frac{d^2y}{dx^2} \; at \; (-2,0) = \frac{-2-32}{1}$$
=-34

12.Let $S = \{(\lambda, \mu) \in R \times R : f(t) = (|\lambda| e^{|\lambda|} - \mu).\ \sin(2|t|), t \in R,$ is a differentiable function$\}$.

Then S is a subset of :

A. R × [0, ∞)
B. [0, ∞) × R
C. R × [-∞, 0)
D. [-∞, 0) × R

Answer ||| A

Solution |||

$$f(t) = \{(|\lambda| e^{|t|} - \mu) \cdot \sin(2|t|)\}$$

$$RHD = \lim_{h \to 0} \frac{f(0+h) - 0}{h}$$

$$= \lim_{h \to 0} (|\lambda| e^h - \mu) \frac{\sin 2h}{h} \times \frac{2}{2}$$

$$= \lim_{h \to 0} \frac{\sin 2h}{2h} \times (|\lambda| e^h - \mu) \times 2$$

$$= 2(|\lambda| e^h - \mu)$$

$$LHD = \lim_{h \to 0} \frac{f(0-h) - 0}{-h}$$

$$= \lim_{h \to 0} \frac{(|\lambda| e^h - \mu) \sin 2h}{-h} \times \frac{2}{2}$$

$$= \lim_{h \to 0} \frac{\sin 2h}{-2h} \times (|\lambda| e^h - \mu) \times 2$$

$$= -2(|\lambda| e^h - \mu)$$

Now LHD = RHD = 0

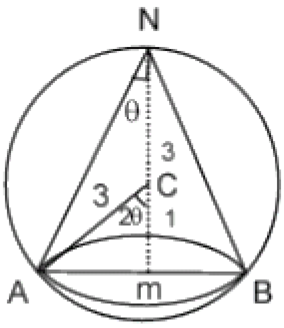
$$\Rightarrow |\lambda| e^h = \mu$$

$$\Rightarrow |\lambda| = \mu \;\; \Rightarrow \mu \geq 0 \;\&\; \lambda \in R$$

13.If a right circular cone, having maximum volume, is inscribed in a sphere of radius 3 cm, then the curved surface area (in cm^2) of this cone is :

A. $6\sqrt{2}\,\pi$
B. $6\sqrt{3}\,\pi$
C. $8\sqrt{2}\,\pi$
D. $8\sqrt{3}\,\pi$

Answer ||| D

Solution ||| 

$$V=\frac{1}{3}\pi r^2h$$

Where r is the radius and h is the height

$$V=\frac{1}{3}\pi(3\sin 2\theta)^2(3+3\cos 2\theta)$$

$$V=72\pi sin^2\theta cos^4\theta$$

$$\frac{dV}{d\theta}=72\pi[2\sin\theta\, cos^5\theta-4sin^3\theta cos^3\theta]=0$$

$$\tan\theta=\frac{1}{\sqrt{2}}$$

Hence curved surface area

$$S=\pi rl$$

$$\pi r\sqrt{(3+3\cos 2\theta)^2+(3\sin 2\theta)^2}$$

$$=18\pi(2\sin\theta\, cos^2\theta)=36\pi.\frac{1}{\sqrt{3}}.\frac{2}{3}=8\sqrt{3}\pi$$

14.If $f\left(\frac{x-4}{x+2}\right)=2x+1,\left(x\in R-\{1,-2\}\right)$, then $\int f(x)\,dx$ is equal to :

(where C is a constant of integration)p

A. $12\log_e|1-x|+3x+C$

B. $-12\log_e|1-x|-3x+C$

C. $12\log_e|1-x|-3x+C$

D. $-12\log_e|1-x|+3x+C$

Answer ||| B

Solution ||| $f\left(\frac{x-4}{x+2}\right)=2x+1$

First, we will convert x in terms of y,

Now Let's assume $\frac{x-4}{x+2}$ = y

=> yx + 2y = x - 4

=> (y-1) x = -2y-4

=> $x = \frac{-2y-4}{y-1}$

Now, we will switch the equation in RHS and LHS, equivalent will remain same.

$$\frac{-2y-4}{y-1} = 2\left\{1 - \frac{3(y+1)}{y-1}\right\}$$

Thus,

$$f(x) = 2\left\{1 - \frac{3(x+1)}{x-1}\right\} + 1$$

$= 3 - \frac{6x+6}{x-1} = \frac{-3x-9}{x-1}$

$$=> f(x) = \frac{3(x+3)}{1-x}$$

$$\int f(x)dx = 3\int \frac{x+3}{1-x}dx = 3\int \frac{(4-(1-x))}{1-x}dx = 3\int \frac{4}{1-x}dx - \int dx$$

$$= -12\ln|1-x| - 3x + C$$

15.The area (in sq. units) of the region $\{x \in : x \geq 0, y \geq 0, y \geq x-2 \text{ and } y \leq \sqrt{x}\}$, is :

A. $\frac{13}{3}$

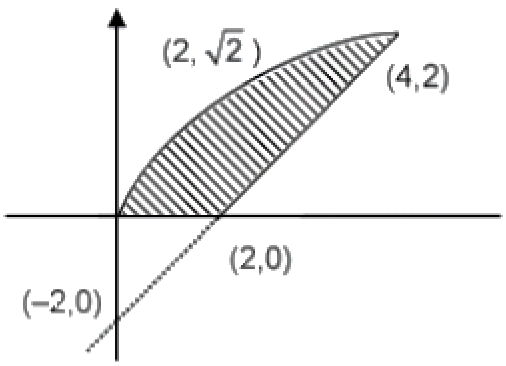
B. $\frac{8}{3}$

C. $\frac{10}{3}$

D. $\frac{5}{3}$

Answer ||| C

Solution |||



$= \int_0^2 \sqrt{x}dx + \int_2^4 (\sqrt{x} - x + 2)dx$

The area under $\sqrt{x}$ can be calcualated directly from 0 to 4. Thus separating the integrals, we have

$=\int_0^4 \sqrt{x}dx + \int_2^4 (-x+2)dx$

$=\frac{2}{3}.8 + (-\frac{x^2}{2} + 2x)_2^4$

$=\frac{16}{3} - 8 + 8 + 2 - 4$

$=\frac{10}{3}$

16.The value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^4x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx$ is :

A. 0

B. $\frac{3}{4}$

C. $\frac{3}{8}\pi$

D. $\frac{3}{16}\pi$

Answer ||| C

Solution ||| We know that,

$$\int_a^b f(x) = \int_a^b f(a+b-x)$$

So applying that we, get

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^4x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^4x\left(1+\log\left(\frac{2-\sin x}{2+\sin x}\right)\right)dx$$

Now on adding both we get

$$I + I = 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^4x\, dx$$

$$I = \frac{3}{4}.\frac{1}{2}.\frac{\pi}{2} = \frac{3\pi}{8}$$

17.Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 2y = f(x)$, where $f(x) = \begin{cases} 1, & x \in [0,1] \\ 0, & \text{otherwise} \end{cases}$ If $y(0) = 0$, then $y\left(\frac{3}{2}\right)$ is :

A. $\frac{e^2+1}{2e^4}$

B. $\frac{1}{2e}$

C. $\frac{e^2-1}{e^3}$

D. $\frac{e^2-1}{2e^3}$

Answer ||| D

Solution ||| At y(0)=0

$$\frac{dy}{dx} + 2y = 1$$

$$\frac{dy}{dx} = 1 - 2y$$

$$\frac{dy}{1-2y} = dx$$

$$x = \frac{\log(1-2y)}{-2} + C$$

So, C=0

At x=3/2

$$\frac{dy}{dx} + 2y = 1 + 0$$

$$\frac{dy}{dx} + 2y = f(x)$$

$$IF = e^{2x}$$

$$(y.e^{2x}) = \int_0^3 f(x)e^{2x}dx$$

$$(y.e^{2x}) = \int_0^1 e^{2x}dx + \int_1^3 0dx$$

$$\frac{e^2-1}{2e^3}$$

18.In a triangle ABC, coordinates of A are (1, 2) and the equations of the medians through B and C are respectively, $x + y = 5$ and $x = 4$. Then area of ΔABC (in sq. units) is :

A. 12

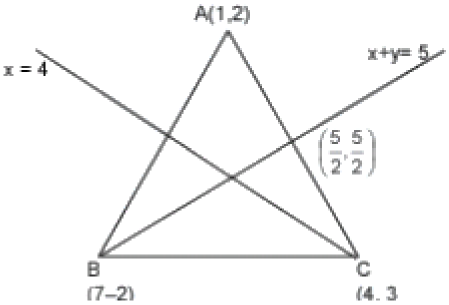
B. 4

C. 5

D. 9

Answer ||| D

Solution |||



Let's assume the meeting point of given median is M

x + y = 5

x = 4

=> y = 1

Thus, M: (4,1)

Point C can be assumed as (4,y)

The midpoint of AC will lie on the median through B

The midpoint of AC = $\left(\frac{1+4}{2}, \frac{2+y}{2}\right)$

It lies on the line x+y = 5

$$\left(\frac{1+4}{2} + \frac{2+y}{2}\right) = 5$$

$$\Rightarrow 5 + 2 + y = 10$$

$$\Rightarrow y = 3$$

So, C : (4,3)

Now, the centroid is M (4,1)

Let's assume coordinate of B be (p,q)

$$(4,1) = \left(\frac{1+4+p}{3}, \frac{2+3+q}{3}\right)$$

=> p = 7 and q = -2

Thus Area=$\frac{1}{2}.\begin{vmatrix}1 & 2 & 1\\ 7 & -2 & 1\\ 4 & 3 & 1\end{vmatrix}$

=9 sq. unit

19.A circle passes through the points (2, 3) and (4, 5). If its centre lies on the line, $y - 4x + 3 = 0$, then its radius is equal to :

A. 2

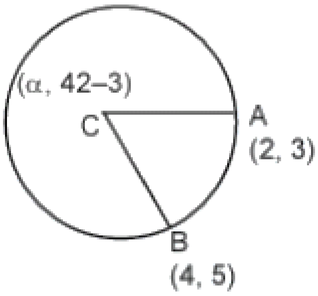
B. $\sqrt{5}$

C. $\sqrt{2}$

D. 1

Answer ||| A

Solution |||



Let (h,k) be the centre

Since, it lies on the line $y - 4x + 3 = 0,$

Thus $k - 4h + 3 = 0$....(1)

Now the BC and AC are equal as they are the radius,

Thus,

$$(h-2)^2 + (k-3)^2 = (h-4)^2 + (k-5)^2$$
$$h^2 + k^2 + 4 - 4h + 9 - 6k = h^2 + k^2 + 16 - 8h + 25 - 10k$$
$$4h + 6k - 13 = 8h + 10k - 41$$
$$4h + 4k = 28$$
$$h + k = 7 \ldots 2$$

Solving 1 and 2 simultaneously we, get

$$k = 5, h = 2$$

Radius=$\sqrt{(h-2)^2 + (k-3)^2} = 2$

20.Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, then the equation of the common tangent to the two parabolas is :

A. $4(x + y) + 3 = 0$

B. $3(x + y) + 4 = 0$

C. $8(2x+y)+3=0$

D. $x+2y+3=0$

Answer ||| A

Solution ||| The length of the latus rectum of each parabola is 3

Thus, the equation of the two parabolas are $y^2 = 3x$ and $x^2 = 3y$

Then, the equation of first parabola be $y = mx + \frac{3}{4m}$

Which is also tangent to $x^2 = 3y$

$$x^2 = 3mx + \frac{9}{4m}$$

$$4mx^2 - 12m^2x - 9 = 0$$

$D = 0$ since the case is tangential.

$$144m^4 = 4(4m)(-9)$$

$$m^4 + m = 0$$

$$m = \{0, -1\}$$

$$Hence, common\, tangent$$

$$y = -x - \frac{3}{4}$$

$$4(x+y) + 3 = 0$$

21.If the tangents drawn to the hyperbola $4y^2 = x^2+1$ intersect the co-ordinate axes at the distinct points A and B, then the locus of the midpoint of AB is :

A. $x^2 - 4y^2 + 16x^2y^2 = 0$

B. $x^2 - 4y^2 - 16x^2y^2 = 0$

C. $4x^2 - 4y^2 + 16x^2y^2 = 0$

D. $4x^2 - 4y^2 - 16x^2y^2 = 0$

Answer ||| B

Solution ||| Let tangent to the hyperbola at point (x_1,y_1) be $4yy_1=xx_1+1$

This tangent intersect coordinate axes at points A and B

Thus A $\left(\frac{-1}{x_1}, 0\right)$ and $B\left(0, \frac{1}{4y_1}\right)$

Let M be the mid-point

Since P(x2,y2) also lies on the hyperbola

$$4{y_2}^2 = {x_2}^2 + 1$$

$$4.\left(\frac{1}{8k}\right)^2 = \left(-\frac{1}{2h}\right)^2 + 1$$

$$4h^2 = 16k^2(1+4h^2)$$
$$x^2 - 4y^2 - 16x^2y^2 = 0$$

22.If β is one of the angles between the normals to the ellipse, $x^2 + 3y^2 = 9$ at the points $\left(3\cos\theta, \sqrt{3}\sin\theta\right)$ and $\left(-3\sin\theta, \sqrt{3}\cos\theta\right)$; $\theta \in \left(0, \frac{\pi}{2}\right)$; then $\frac{2\cot\beta}{\sin 2\theta}$ is equal to :

A. $\frac{2}{\sqrt{3}}$

B. $\frac{1}{\sqrt{3}}$

C. $\sqrt{2}$

D. $\frac{\sqrt{3}}{4}$

Answer ||| A

Solution ||| $\frac{x^2}{9} + \frac{y^2}{3} = 1$

a = 3 and b = $\sqrt{3}$

Equation of normal to standard ellipse at point (a cosθ, b sinθ)

= ax sec θ - by cosec θ = $a^2 - b^2$

Thus, $Normal\ at\ \left(3\cos\theta, \sqrt{3}\sin\theta\right)$ is

$$3\sec\theta\,.\,x - \sqrt{3}cosec\ \theta y = 6$$

And, $Normal\ at\ \left(-3\sin\theta\,, \sqrt{3}\cos\theta\right)$ is

−3cosecθ .x − 3 secθ y = 6

Let the angle between two normal is β

Thus, $\tan\beta = \left|\frac{\sqrt{3}\tan\theta + \sqrt{3}\cot\theta}{1-3}\right| = \left|-\frac{\sqrt{3}}{2\sin\theta\cos\theta}\right|$

$$\frac{2\cot\beta}{\sin 2\theta} = \frac{2}{\sqrt{3}}$$

23.A variable plane passes through a fixed point (3, 2, 1) and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to yz-plane through A, a second plane is drawn parallel zx-plane through B and a third plane is drawn parallel to xy-plane through C. Then the locus of the point of intersection of these three planes, is :

A. $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$

B. $x+y+z=6$

C. $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$

D. $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

Answer ||| D

Solution ||| Let plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

It passes through (3,2,1)

$$\frac{3}{a}+\frac{2}{b}+\frac{1}{c}=1$$

Now (a,0,0), (0,b,0) and (0,0,c) are the points of intersection of the plane

Thus, a = x, b = y and c = z

Hence, the locus is

$$\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$$

24.An angle between the plane, $x+y+z=5$ and the line of intersection of the planes, $3x+4y+z-1=0$ and $5x+8y+2z+14=0$, is :

A. $\sin^{-1}\left(\sqrt{3/17}\right)$

B. $\cos^{-1}\left(\sqrt{3/17}\right)$

C. $\cos^{-1}\left(3/\sqrt{17}\right)$

D. $\sin^{-1}\left(3/\sqrt{17}\right)$

Answer ||| A

Solution |||

3x + 4y + z - 1 = 0 and 5x + 8y + 2z + 14 = 0 are two given planes.

The two planes will intersect at some line.

The direction ratio of that line can be given by the cross product of normal of the above planes.

Thus,

$$\vec{b}=\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 4 & 1 \\ 5 & 8 & 2 \end{vmatrix}=-\boldsymbol{j}+4\boldsymbol{k}$$

Now, if the direction vector of the line is $\vec{u}$ = (u_1, u_2, u_3) and

the normal vector of the plane is $\vec{n}$ = (A, B , C)

Then, the angle between the line and the plane is given as

$$\alpha = \sin^{-1}\left\{\frac{|A\cdot u_1 + B\cdot u_2 + C\cdot u_3|}{\sqrt{A^2+B^2+C^2}\sqrt{u_1^2+u_2^2+u_3^2}}\right\}$$

Here, $\vec{n}$ = (1,1,1) since plane equation is x + y + z = 5

and $\vec{u}$ = (0,1,-4) since line is -j + 4k

$$\therefore \alpha = \sin^{-1}\left\{\frac{|0-1+4|}{\sqrt{3}\sqrt{17}}\right\}$$

$$= \sin^{-1}\sqrt{\frac{3}{17}}$$

25.If $\bar{a}$, $\bar{b}$, and $\bar{c}$ are unit vectors such that $\bar{a}+2\bar{b}+2\bar{c}=\bar{0}$, then $|\bar{a}\times\bar{c}|$ is equal to :

A. $\frac{\sqrt{15}}{4}$

B. $\frac{1}{4}$

C. $\frac{15}{16}$

D. $\frac{\sqrt{15}}{16}$

Answer ||| A

Solution |||

$$\vec{a}+2\vec{b}+2\vec{c}=0$$
$$\Rightarrow \vec{a}+2\vec{c}-=-2\vec{b}$$
$$\Rightarrow |\vec{a}|^2+4|\vec{c}|^2+4\vec{a}\cdot\vec{c}=4|\vec{b}|^{-2}$$

1+ 4 + 4 cos θ = 4

=> cos θ = -1/4

$$\therefore \sin\theta = \frac{\sqrt{1-\cos^2\theta}}{\sqrt{1-\frac{1}{16}}} = \frac{\sqrt{15}}{4}$$

$$|\vec{a}\times\vec{b}| = |\vec{a}|-|\vec{c}|-\sin\theta$$

$$= \frac{\sqrt{15}}{4}$$

26.The mean of a set of 30 observations is 75. If each observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. Then λ is equal to :

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. $\frac{10}{3}$

Answer ||| C

Solution ||| Suppose M is the mean
$x_1 + x_2 + x_3 .....x_m$ = 75 x 30 = 2250

Now, each observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same.
=>$\lambda(x_1 + x_2 + x_3 .....x_m)$ - 25 x 30 = 75 x 30
=> $\lambda(x_1 + x_2 + x_3 .....x_m)$ = 3000
=> λ x 2250 = 3000
=> $\lambda = \frac{4}{3}$

27.A box ‘A’ contains 2 white, 3 red and 2 black balls. Another box ‘B’ contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box ‘B’ is :

A. $\frac{9}{16}$

B. $\frac{7}{16}$

C. $\frac{9}{32}$

D. $\frac{7}{8}$

Answer ||| B

Solution ||| A->2 white , 3 red , 2 black
B->4 white, 2 red, 3 black
Let E be the event of drawing one white and one red ball

P(E)=$\frac{1}{2}.\left[\frac{2.3}{C(7,2)}+\frac{4.2}{C(9,2)}\right]=\frac{1}{2}\left[\frac{6}{21}+\frac{8}{36}\right]=\frac{1}{2}\left[\frac{2}{7}+\frac{2}{9}\right]=\frac{16}{63}$

P(B/E)=$\frac{P(B)P\left(\frac{E}{B}\right)}{P(E)}=\frac{\frac{1}{9}}{\frac{16}{63}}=\frac{7}{16}$

28.If tanA and tanB are the roots of the quadratic equation, $3x^2-10x-25=0$, then the value of $3\sin^2(A+B)-10\sin(A+B).\ \cos(A+B)-25\cos^2(A+B)$ is :

A. −10

B. 10

C. −25

D. 25

Answer ||| C

Solution ||| tan A and tan B are the roots of $3x^2$ - 10x - 25 = 0

Thus, sum of the roots, $\tan A+\tan B=\frac{10}{3}$

Product of the roots, $\tan B\,.\tan B=-\frac{25}{3}$

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\frac{10}{3}}{1+\frac{25}{3}}$$

$=\frac{5}{14}$

Since sin (A+B) = tan (A+B) x cos(A+B)

So, sin (A+B) = $\frac{5}{\sqrt{221}}$ and cos (A+B) = $\frac{14}{\sqrt{221}}$

Now,

$3\sin^2(A+B)-10\sin(A+B).\ \cos(A+B)-25\cos^2(A+B)$

$$\frac{3.25}{221}-\frac{10.5.14}{221}-25.\frac{14^2}{221}=-25$$

29.An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}$ km above it, is observed at an elevation of 60 from a point on the ground. If, after five seconds, its elevation from the same point, is 30^o, then the speed (in km/hr) of the aeroplane, is :

A. 1500

B. 1440

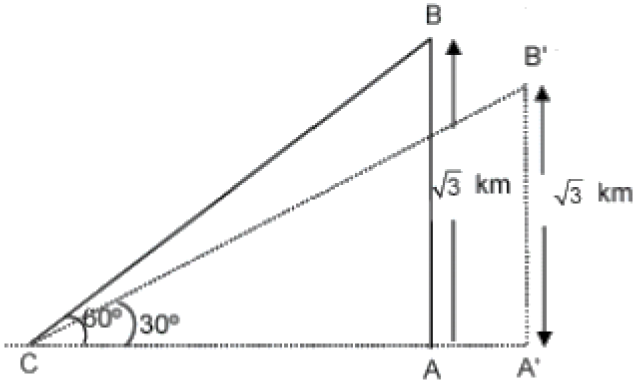
C. 750

D. 720

Answer ||| B

Solution ||| The angle of elevation is 60 degrees from point C.

After a time period of 5 seconds, the aeroplane reaches B, making 30 degrees with C.



In triangle ABC, using the basic concept:

AC=$\sqrt{3}\cot 60 = 1$

$In\ CA'B';\ A'C = \sqrt{3}\cot 30 = 3$

Hence distance AA' = 3 - 1 = 1

$$Speed = \frac{Distance}{Time} = \frac{2}{5}.60.60 = 1440\ kmph$$

30.If $(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$ is false, then the truth values of p, q and r are, respectively :

A. F, T, F

B. T, F, T

C. T, T, T

D. F, F, F

Answer ||| B

Solution ||| (1) (T^F)^(T^T)->(FvT)=T; (F^T)->T=T

(2) (F^F)^(F^F)->TvT=T; F->T=T

(3) (T^T)^(T^T)->FvF=F; T->F=F

(4) (F^T)^(F^F)->TvF

F^F->T

F->T=T