1.The relative error in the determination of the surface area of a sphere is α. Then the relative error in the determination of its volume is:

A. $\frac{3}{2}\alpha$

B. $\frac{2}{3}\alpha$

C. $\frac{5}{2}\alpha$

D. α

Answer ||| A

Solution ||| We know that,

Surface area of sphere is given by S = 4πr^2

$$\therefore Relative\ error\ in\ surface\ area\ \alpha = \frac{\Delta S}{S} = 2\times\frac{\Delta r}{r}\ ...(1)$$

Now,

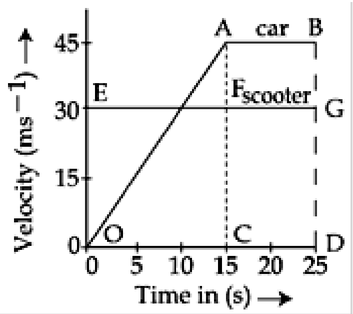
Volume of the sphere is given by V = 4/3 πr^3

$$\therefore Relative\ error\ in\ volume\ \beta = \frac{\Delta V}{V} = 3\times\frac{\Delta r}{r}\ ...(2)$$

$$But\ from\ equation\ (1)\ \frac{\Delta r}{r} = \frac{\alpha}{2}$$

$$\therefore Relative\ error\ in\ volume\ \beta = \frac{\Delta V}{V} = 3\times\frac{\alpha}{2}$$

2.The velocity-time graphs of a car and a scooter are shown in the figure. (i) The difference between the distance travelled by the car and the scooter in 15 s and (ii) the time at which the car will catch up with the scooter are, respectively.



A. 112.5 m and 22.5 s

B. 337.5 m and 25 s

C. 112.5 m and 15 s

D. 225.5 m and 10 s

Answer ||| A

Solution ||| (i)

Area under the velocity time graph gives the value of travelled distance.

So, the distance travelled by the car in 15 seconds

= Area of triangle OAC

= 1/2 × 45 × 15

= 337.5 m

The distance travelled by the scooter in 15 seconds

= Area of rectangle OEFC

= 30 × 15 = 450 m

∴ Difference in distance travelled by the car and scooter in 15 seconds

= 450 – 337.5 = 112.5 m

(ii)

Let the car catches scooter in time t,

So,

337.5 + 45(t−15) = 30t

∴ t = 337.5/15 = 22.5 s

3.A given object takes n times more time to slide down a 45^o rough inclined plane as it takes to slide down a perfectly smooth 45^o incline. The coefficient of kinetic friction between the object and the incline is :

A. $\frac{1}{2-n^2}$

B. $1-\frac{1}{n^2}$

C. $\sqrt{1-\frac{1}{n^2}}$

D. $\sqrt{\frac{1}{1-n^2}}$

Answer ||| B

Solution ||| Let, S be the distance travelled by the object while sliding down the slope on a inclined plane. Let time taken in the rough inclined is t_1. And time taken in smooth inclined is t_2.

So, $t_1 = nt_2$ .....(1)

Acceleration at smooth surface is $a_2 = g\sin\theta$

Acceleration at rough surface is $a_1 = g\sin\theta - \mu g\cos\theta$

So according to equation of motion,

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2s}{a}}$$

So according to equation (1)

$$\sqrt{\frac{2s}{a_1}} = n\sqrt{\frac{2s}{a_2}}$$

$$\frac{1}{a_1} = \frac{n^2}{a_2} \Rightarrow a_1 = \frac{a_2}{n^2}$$

$$g\sin\theta - \mu g\cos\theta = \frac{g\sin\theta}{n^2}$$

$$sin45^o - \mu cos45^o = \frac{sin45^o}{n^2}$$

$$\frac{1}{\sqrt{2}} - \frac{\mu}{\sqrt{2}} = \frac{1}{n^2\sqrt{2}}$$

$$1 - \mu = \frac{1}{n^2}$$

$$\mu = \left(1 - \frac{1}{n^2}\right)$$

4.An automobile, travelling at 40 km/h, can be stopped at a distance of 40 m by applying brakes. If the same automobile is travelling at 80 km/h, the minimum stopping distance, in metres, is (assume no skidding):

A. 75 m

B. 100 m

C. 150 m

D. 160 m

Answer ||| D

Solution ||| We know that,

V^2 = 2as

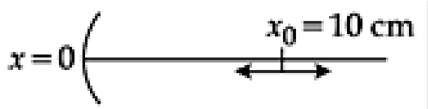
We will use this condition in the given two cases to arrive at the final answer.

$$\therefore \frac{V_1^2}{V_2^2} = \frac{s_1}{s_2}$$

$$\therefore \frac{40^2}{80^2} = \frac{40}{s_2}$$

$\therefore s_2 = 160$ m.

5.A particle is oscillating on the X-axis with amplitude 2 cm about the point $x_0 = 10$ cm, with a frequency ω. A concave mirror of focal length 5 cm is placed at the origin (see figure).



Identify the correct statements.

1) The image executes periodic motion.

2) The image executes non-periodic motion.

3) The turning points of the image are asymmetric w.r.t. the image of the point at $x = 10$ cm.

4) The distance between the turning points of the oscillation of the image is $\frac{100}{21}$ cm.

A. (1), (4)

B. (1), (3), (4)

C. (2), (4)

D. (2), (3)

Answer ||| B

Solution ||| As the object oscillates, the image will also oscillate and as a result the image will show a periodic motion.

When the object is at 8 cm i.e. (X_0− 2 cm), (using mirror formula)

$$v_1 = \frac{f \times u}{u - f} = \frac{5 \times 8}{8 - 5} = \frac{40}{3}\ cm$$

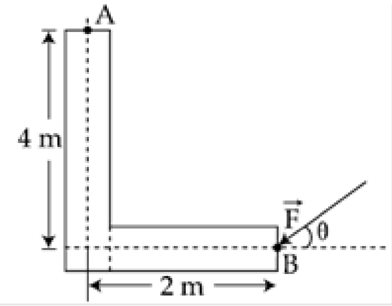
When the object is at 12 cm i.e. (X_0 + 2 cm),

$$v_2 = \frac{f \times u}{u - f} = \frac{5 \times 12}{12 - 5} = \frac{60}{7}\ cm$$

Distance between two images = $|v_1 - v_2|$ = 100/21 cm

Hence, 1, 3, 4 are correct.

6.A force of 40 N acts on a point B at the end of an L-shaped object, as shown in the figure. The angle θ that will produce maximum moment of the force about point A is given by :



A. $\tan\theta = \frac{1}{2}$

B. $\tan\theta = 2$

C. $\tan\theta = 4$

D. $\tan\theta = \frac{1}{4}$

Answer ||| A

Solution ||| Moment at point A due to Force F=40 N will be:

M_A = (40cosθ × 4) + (40sinθ × 2)

Now, for moment to be maximum, $\left(\frac{dM_a}{d\theta}\right) = 0;$

∴ (−40sinθ × 4) + (40cosθ × 2) = 0

∴ 4sinθ = 2cosθ

∴ tanθ = 1/2

7.A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$, and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is :

A. $-\frac{GMm}{2R}$

B. $+\frac{GMm}{6R}$

C. $\frac{GMm}{2R}$

D. $-\frac{GMm}{6R}$

Answer ||| D

Solution ||| $Initial\ energy\ of\ the\ body\ E_i = -\frac{GmM}{2R}$

After splitting, one half of the mass moves in orbit of radius R/2 and other half moves in the orbit of 3R/2.

So,

$$Final\ energy\ of\ the\ body\ after\ spiliting\ E_f = -\frac{\frac{GMm}{2}}{2\left(\frac{R}{2}\right)} - \frac{\frac{GMm}{2}}{2\left(\frac{3R}{2}\right)}$$

$$= -\frac{GmM}{2R} - \frac{GmM}{6R} = -\frac{2GmM}{3R}$$

Difference between final and initial energies:

$$E_f - E_i = -\frac{2GmM}{3R} + \frac{GmM}{2R} = -\frac{GmM}{6R}$$

8.Take the mean distance of the moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22} kg and 2×10^{30} kg respectively. The radius of the earth is 6400 km. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest points on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. Then, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is:

A. 2

B. 10^{-2}

C. 0.6

D. 6

Answer ||| A

Solution ||| Let force acting between earth and moon be,

$$F_1 = \frac{GM_e m}{r_1^2}$$

Let force acting between earth and sun be,

$$F_2 = \frac{GM_e M_s}{r_2^2}$$

$$\Delta F_1 = -\left(\frac{2GM_e m}{r_1^3}\right)\Delta r_1 \ \&\ \Delta F_2 = -\left(\frac{2GM_e M_s}{r_2^3}\right)\Delta r_2$$

Now,

$$\frac{\Delta F_1}{\Delta F_2} = \left(\frac{m\Delta r_1}{r_1^3}\right)\left(\frac{r_2^3}{M_s \Delta r_2}\right) = \left(\frac{m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right) \dots (1)$$

Now, using the data of the question

$$\Delta r_1 = \Delta r_2 = 2R_{Earth}$$

m = 8×10^{22} kg

M_S = 2×10^{30} kg

r_1 = 0.4×10^6 km

r_2 = 150×10^6 km

Putting the above values in equation (1) we get,

$$\frac{\Delta F_1}{\Delta F_2} = 2$$

9.A thin uniform tube is bent into a circle of radius r in the vertical plane. Equal volumes of two immiscible liquids, whose densities are ρ_1 and $\rho_2 (\rho_1 > \rho_2)$, fill half the circle. The angle θ between the radius vector passing through the common interface and the vertical is :

A. $\theta = \tan^{-1}\left(\frac{\rho_1}{\rho_2}\right)$

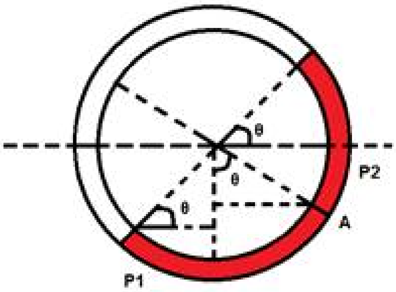
B. $\theta = \tan^{-1}\left(\frac{\rho_2}{\rho_1}\right)$

C. $\theta = \tan^{-1}\left[\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)\right]$

D. $\theta = \tan^{-1}\left(\frac{\rho_1 + \rho_2}{\rho_1 - \rho_2}\right)$

Answer ||| C

Solution |||



From figure, equating pressure at point A

$$\rho_1 gR(\cos\theta - \sin\theta) = \rho_2 gR(\cos\theta + \sin\theta)$$

$$\frac{\rho_1}{\rho_2} = \frac{(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)} = \frac{\tan\theta + 1}{1 - \tan\theta}$$

$$(\rho_1 + \rho_2)\tan\theta = \rho_1 - \rho_2$$

$$\therefore \theta = \tan^{-1}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)$$

10.A Carnot’s engine works as a refrigerator between 250 K and 300 K. It receives 500 cal heat from the reservoir at the lower temperature. The amount of work done in each cycle to operate the refrigerator is:

A. 420 J
B. 772 J
C. 2100 J
D. 2520 J

Answer ||| A

Solution ||| The coefficient of performance for a refrigerator is

$$\beta = \frac{Q_2}{W}$$

$$\frac{T_2}{T_1 - T_2} = \frac{Q_2}{W}$$

$$\frac{W}{Q_2} = \frac{T_1 - T_2}{T_2}$$

$$\frac{W}{500} = \frac{300 - 250}{250}$$

$$W = \frac{50}{250} \times 500$$

$$W = 100 \text{ cal}$$

$$W = 100 \times 4.2$$

$$W = 420 \text{ J}$$

11.One mole of an ideal monoatomic gas is compressed isothermally in a rigid vessel to double its pressure at room temperature, 27°C. The work done on the gas will be :

A. 300 R
B. 300 R ln 6
C. 300 R ln 2
D. 300 R ln 7

Answer ||| C

Solution ||| Let's assume initial pressure to be P_1

Thus, the final pressure is $P_2 = 2P_1$ (Since the pressure has doubled)

Work done in isothermal process:

$$W = mRT \ln\left(\frac{P_2}{P_1}\right)$$

=> W = 1 x R x 300 x ln $\left(\frac{2P_1}{P_1}\right)$

=> W = 300 R ln2 (Since, $P_2 = 2P_1$)

12.A body of mass M and charge q is connected to a spring of spring constant k. It is oscillating along x-direction about its equilibrium position, taken to be at $x = 0$, with an amplitude A. An electric field E is applied along the x-direction. Which of the following statements is correct?

A. The new equilibrium position is at a distance $\frac{qE}{2K}$ from $x = 0$.

B. The total energy of the system is $\frac{1}{2}m\omega^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}$.

C. The total energy of the system is $\frac{1}{2}m\omega^2A^2 - \frac{1}{2}\frac{q^2E^2}{k}$.

D. The new equilibrium position is at a distance $\frac{2qE}{K}$ from $x = 0$.

Answer ||| B

Solution ||| The equilibrium position will shift to the point where the resultant force will be zero.

$$\therefore kx_{equlibrium} = qE \Rightarrow x_{equlibrium} = \frac{qE}{k}$$

$$\text{Energy} = \frac{1}{2}m\omega^2\left[A^2 + x_{equlibrium}^2\right] = \frac{1}{2}m\omega^2\left[A^2 + \left(\frac{qE}{k}\right)^2\right] =$$

$$\frac{1}{2}m\omega^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}.$$

13.A tuning fork vibrates with frequency 256 Hz and gives one beat per second with the third normal mode of vibration of an open pipe. What is the length of the pipe? (Speed of sound in air is 340 ms^{-1})

A. 220 cm

B. 190 cm

C. 180 cm

D. 200 cm

Answer ||| D

Solution ||| As the tuning fork vibrates with 256 Hz frequency and gives 1 beat per second,

The frequency of organ pipe
= (Tuning fork frequency + 1 Hz) or (Tuning fork frequency -1 Hz)
=257 Hz or 255 Hz.

$$\frac{3v}{2L}=255, \text{ or } \frac{3v}{2L}=257 \qquad \text{(For 3rd Normal mode of vibration)}$$

$$L=\frac{3v}{2\times255}, \text{ or } L=\frac{3v}{2\times257}$$

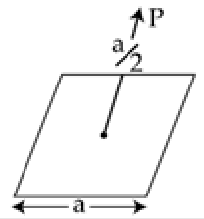
$$L=\frac{3\times340}{2\times255}, \text{ or } L=\frac{3\times340}{2\times257}$$

$$L=2, \text{ or } 1.98$$

From the above data, we can say that

L≈ 2m = 200 cm

14.A charge Q is placed at a distance a/2 above the centre of the square surface of edge a as shown in the figure



The electric flux through the square surface is :

A. $\frac{Q}{\epsilon_0}$

B. $\frac{Q}{2\epsilon_0}$

C. $\frac{Q}{3\epsilon_0}$

D. $\frac{Q}{6\epsilon_0}$

Answer ||| D

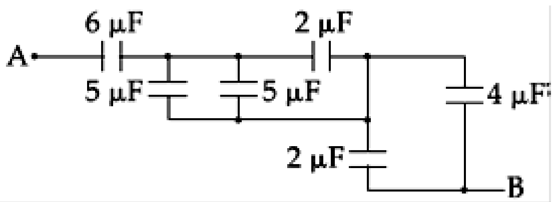
Solution ||| Here, we can consider the particle to be on the center of a cube of length a. Moreover, the given surface represents one face of the cube. The cube has 6 sides.

So, according to Gauss's law, the flux associated with the face of the cube is given by:

$\Phi_e = \frac{Q}{6\epsilon_0}$

This is because the flux will be equally associated with all other 6 sides.

15.The equivalent capacitance between A and B in the circuit given below, is :



A. 2.4 μF

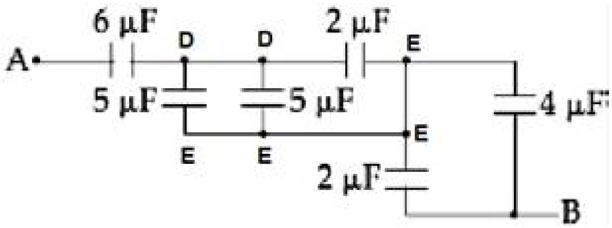
B. 4.9 μF

C. 3.6 μF

D. 5.4 μF

Answer ||| A

Solution |||



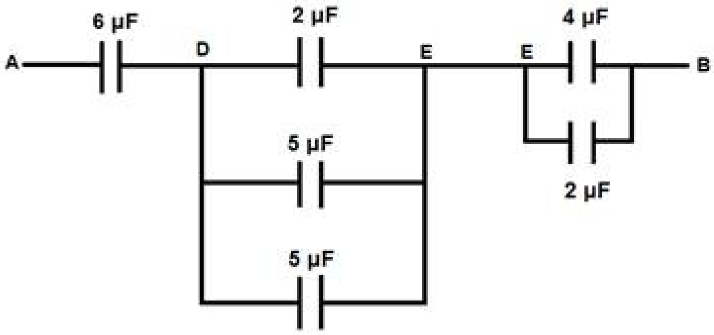
The first step would include simplification of the circuit.

In the above image, two points are marked D, and four points are marked as E.

D and E points are at the same potential, thus all the resistors between D and E are in parallel.

Similarly, resistors between E and B are in parallel

Thus, the simplified diagram would be



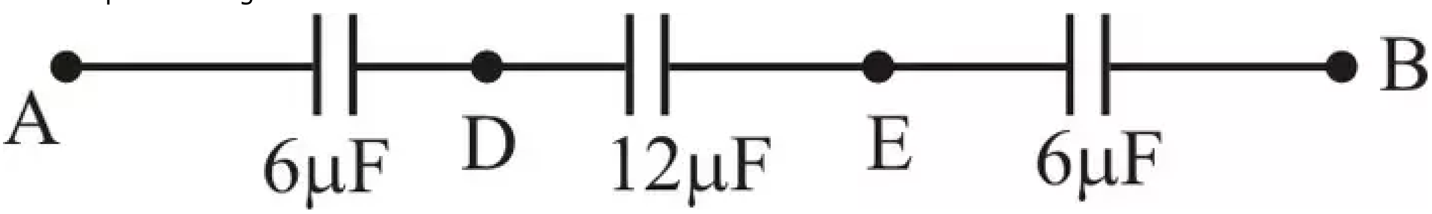
Now adding all the capacitors in parallel using formula,

$C_{eq} = C_1 + C_2 + C_3 + ... + C_n$

Equivalent capacitance between D and E; C_{DE} = (2 + 5 + 5)μF = 12μF

Similarly, the equivalent capacitance between B and E = (4+ 2) μF = 6μF

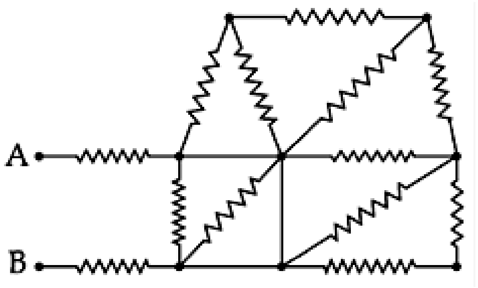
The simplified image would be:



Now, the equivalent capacitance of above-simplified circuit would be:

$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12} \Rightarrow C_{eq} = \frac{12}{5} = 2.4\ \mu F$$

16.In the given circuit all resistances are of value R ohm each. The equivalent resistance between A and B is :



A. 2R

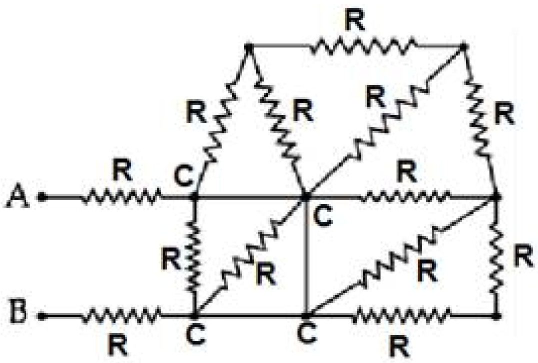
B. 3R

C. $\frac{5R}{3}$

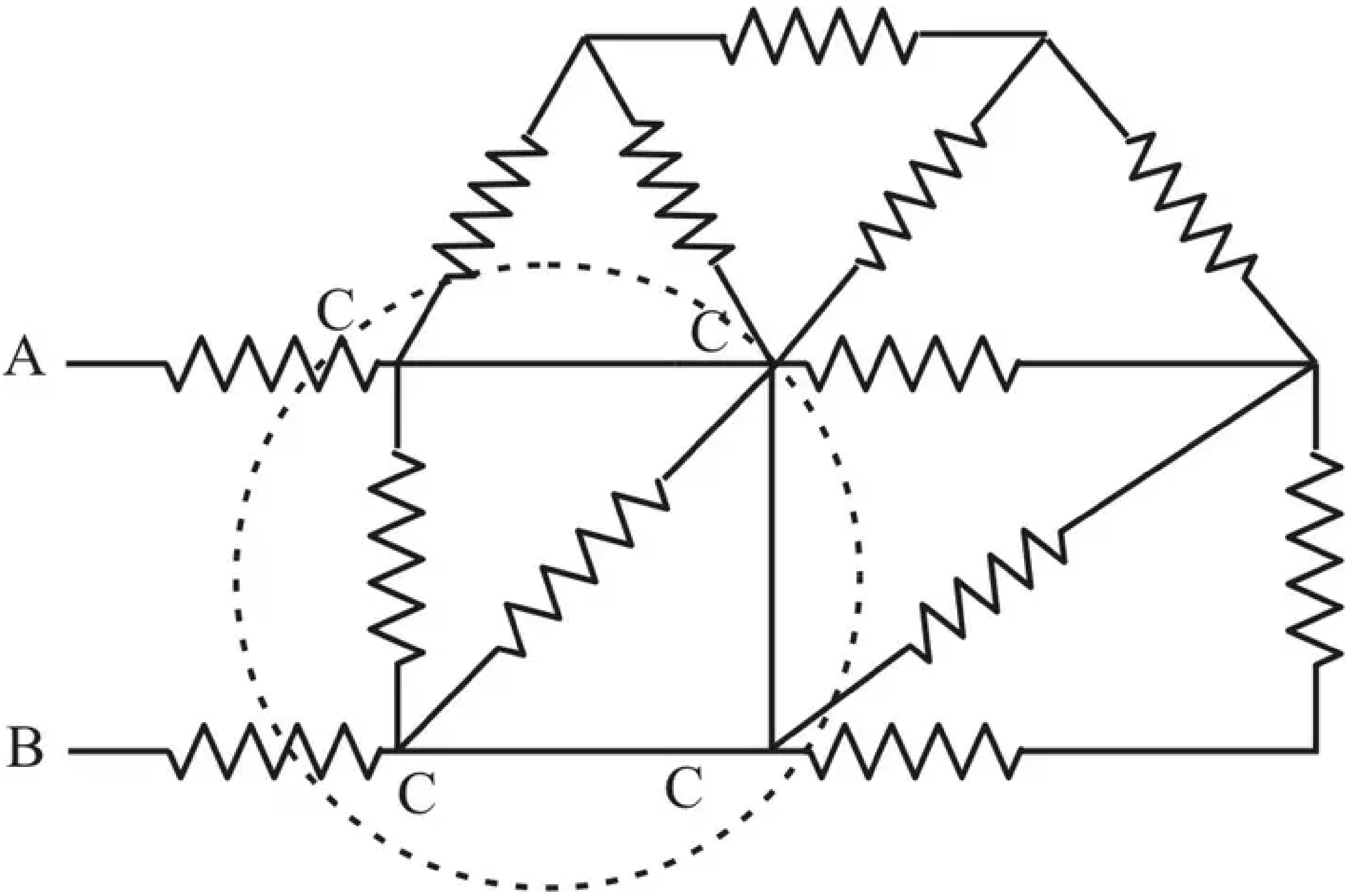
D. $\frac{5R}{2}$

Answer ||| A

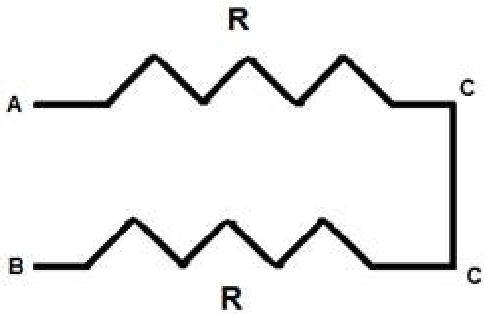
Solution |||



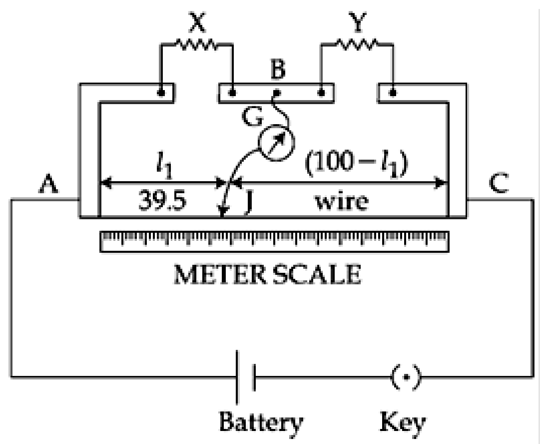
At point C's circuit is short-circuited, so current will not flow through these resistances



Thus, the equivalent circuit is



$\therefore R_{eq} = R+R = 2R$

17. 

In a meter bridge, as shown in the figure, it is given that resistance Y=12.5 Ω and that the balance is obtained at a distance 39.5 cm from end A (by Jockey J). After interchanging the resistances X and Y, a new balance point is found at a distance l_2 from end A. What are the values of X and l_2 ?

A. 8.16 Ω and 60.5 cm

B. 19.15 Ω and 39.5 cm

C. 8.16 Ω and 39.5 cm

D. 19.15 Ω and 60.5 cm

Answer ||| A

Solution ||| For a balanced meter bridge,

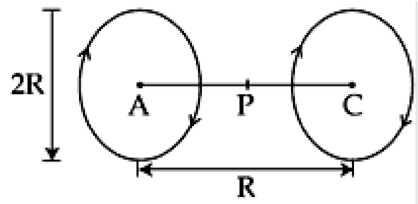
X × (100 – 39.5) = Y × 39.5

∴ X = (12.5 × 39.5)/60.5 = 8.16 Ω

Now, if we interchange X and Y, then the L_1 and (100 – L_1) will also get interchanged.

So, L_2 = (100 – 39.5) = 60.5 cm

18.A Helmholtz coil has a pair of loops, each with N turns and radius R. They are placed coaxially at distance R and the same current I flows through the loops in the same direction. The magnitude of magnetic field at P, midway between the centres A and C, is given by [Refer to figure given below] :



A. $\frac{8\ N\ \mu_0 I}{5^{1/2} R}$

B. $\frac{8\ N\ \mu_0 I}{5^{3/2} R}$

C. $\frac{4\ N\ \mu_0 I}{5^{1/2} R}$

D. $\frac{4\ N\ \mu_0 I}{5^{3/2} R}$

Answer ||| B

Solution ||| We know that the magnetic field at axis at a distance x from center O of a current carrying circular coil is:

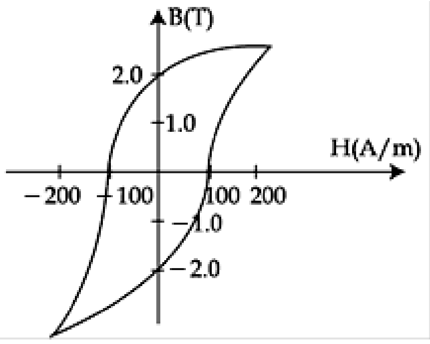
$$B = \frac{\mu_o n I r^2}{2(r^2 + x^2)^{\frac{3}{2}}}$$

Here, x = R/2 and B′ = B+B = 2B

$$B' = 2\left(\frac{\mu_o n I R^2}{2(R^2 + (R/2)^2)^{\frac{3}{2}}}\right) = 2\left(\frac{\mu_o n I R^2}{2\left(R^2 + \frac{R^2}{4}\right)^{\frac{3}{2}}}\right) = \frac{\mu_o n I R^2}{\frac{5^{3/2}}{8}}$$

$$= \frac{8\mu_o n I R^2}{5^{3/2}}$$

19.The B-H curve for a ferromagnet is shown in the figure. The ferromagnet is placed inside a long solenoid with 1000 turns/cm. The current that should be passed in the solenoid to demagnetise the ferromagnet completely is :



A. 1 mA
B. 2 mA
C. 20 μA
D. 40 μA

Answer ||| A

Solution ||| According to the given data

n = 1000 turns/cm = 10^5 turns/m
Here from the B-H curve, Coercivity of Ferro magnet H = 100 A/m as the curve cuts the H axis at 100 A/M.

∴ nI = 100
∴ I = $100/10^5$ = 1 mA

20.An ideal capacitor of capacitance 0.2 μF is charged to a potential difference of 10 V. The charging battery is then disconnected. The capacitor is then connected to an ideal inductor of self inductance 0.5 mH. The current at a time when the potential difference across the capacitor is 5 V, is :

A. 0.34 A
B. 0.25 A
C. 0.17 A
D. 0.15 A

Answer ||| C

Solution ||| In the complete process, no energy is lost. Thus, we can use the law of conservation of energy.

According to the law of conservation of energy.
Energy in capacitor at 10V = Energy in capacitor at 5V + Energy in inductor at 5V

$$\frac{1}{2}CV_1^2 = \frac{1}{2}CV_2^2 + \frac{1}{2}LI^2$$

$$\frac{1}{2}\times0.2\times10^{-6}\times(10)^2=\frac{1}{2}\times0.2\times10^{-6}\times(5)^2+\frac{1}{2}\times0.5\times10^{-3}\times(I)^2$$

$\therefore$ I = 0.17 A

21.A monochromatic beam of light has a frequency $\nu = \frac{3}{2\pi}\times10^{12}\,Hz$ and is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$. It is polarized along the $\hat{k}$ direction. The acceptable form for the magnetic field is :

A. $k\frac{E_0}{C}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)\cos\left[10^4\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right).\vec{r}-(3\times10^{12})t\right]$

B. $\frac{E_o}{C}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}\cos\left[10^4\frac{(\hat{i}-\hat{j})}{\sqrt{2}}\cdot\bar{r}-(3\times10^{12})t\right]$

C. $\frac{E_o}{C}\hat{k}\cos\left[10^4\frac{(\hat{i}+\hat{j})}{\sqrt{2}}\cdot\bar{r}+(3\times10^{12})t\right]$

D. $\frac{E_o}{C}\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}\cos\left[10^4\frac{(\hat{i}+\hat{j})}{\sqrt{2}}\cdot\bar{r}-(3\times10^{12})t\right]$

Answer ||| A

Solution ||| The direction of the magnetic field is,

$\Rightarrow \widehat{E}\times\widehat{k}$

$\Rightarrow \frac{(\widehat{i}+\widehat{j})}{\sqrt{2}}\times\widehat{k}$

$\Rightarrow \frac{(-\widehat{j}+\widehat{i})}{\sqrt{2}}$

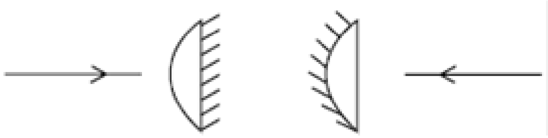
$\Rightarrow \frac{(\widehat{i}-\widehat{j})}{\sqrt{2}}$

The polarization is along the direction of $\widehat{k}$.

This condition is only satisfied by equation (1).

22.A planoconvex lens becomes an optical system of 28 cm focal length when its plane surface is silvered and illuminated from left to right as shown in Fig-A.

If the same lens is instead silvered on the curved surface and illuminated from other side as in Fig.B, it acts like an optical system of focal length 10 cm. The refractive index of the material of lens is :



A. 1.50

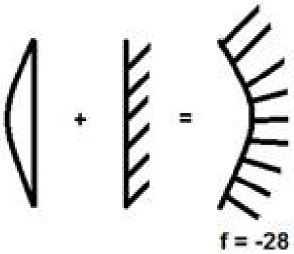
B. 1.55

C. 1.75

D. 1.51

Answer ||| B

Solution ||| Case (a):

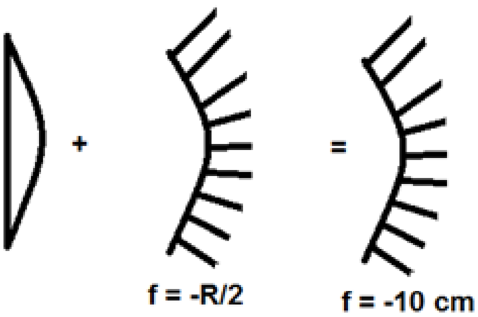


$$\frac{1}{f_1} = \left(\frac{\mu - 1}{R}\right)$$

P = 2P1 + P2

$$\frac{1}{28} = 2\left(\frac{\mu - 1}{R}\right) \ ...(1)$$

Case (b):



$$P = 2P_1 + P_2 \Rightarrow \frac{1}{10} = 2\left(\frac{\mu - 1}{2}\right) + \frac{2}{R}$$

$$\frac{1}{10} = 2\left(\frac{1}{28}\right) + \frac{2}{R} \Rightarrow R = \frac{280}{9}\ cm$$

Putting the value of R in equation (1)

$$\frac{1}{28} = 2\left(\frac{\mu - 1}{280/9}\right) \Rightarrow (\mu - 1) = \frac{5}{9} \Rightarrow \mu = 1.55$$

23.Light of wavelength 550 nm falls normally on a slit of width 22 × 10^{-5} cm. The angular position of the second minima from the central maximum will be (in radians) :

A. π/12

B. π/8

C. π/6

D. π/4

Answer ||| B

Solution ||| At any minima, destructive interference occurs.

For destructive interference,

$d\sin\theta=(m+1/2)\lambda$, for $m=0,1,-1,2,-2$

Here, we are asked about second minima;

Thus, m = 1

Hence,

$\sin\theta = \dfrac{3\lambda}{2d}$

$$= \frac{3\times550\times10^{-9}}{2\times22\times10^{-7}} = 0.375$$

$$\Rightarrow \theta = \sin^{-1}(0.375) \approx \frac{\pi}{8}\ rad$$

24.Two electrons are moving with non-relativistic speeds perpendicular to each other. If corresponding de Broglie wavelengths are λ_1 and λ_2, their de Broglie wavelength in the frame of reference attached to their centre of mass is :

A.

$\lambda_{CM} = \lambda_1 = \lambda_2$

B. $\lambda_{CM} = \dfrac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$

C. $\dfrac{1}{\lambda_{CM}} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$

D. $\lambda_{CM} = \left(\dfrac{\lambda_1+\lambda_2}{2}\right)$

Answer ||| B

Solution ||| Let momentum of each particle be:

$$p_1 = \frac{h}{\lambda_1}\hat{\imath} \,\&\, p_2 = \frac{h}{\lambda_2}\hat{\jmath}\ [They\ are\ moving\ perpendicular\ to\ each\ other]$$

Velocity of center of mass of the two electron system will be:

$$V_{cm} = \frac{h}{2m\lambda_1}\hat{\imath} + \frac{h}{2m\lambda_2}\hat{\jmath}$$

$$\therefore |V_{cm}| = \sqrt{\left(\frac{h}{2m\lambda_1}\right)^2 + \left(\frac{h}{2m\lambda_2}\right)^2}$$

$$\lambda_{cm} = \frac{h}{p_{cm}} = \frac{h}{mv_{cm}} = \frac{h}{m\sqrt{\left(\frac{h}{2m\lambda_1}\right)^2 + \left(\frac{h}{2m\lambda_2}\right)^2}} \Rightarrow \lambda_{CM} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

25.The energy required to remove the electron from a singly ionized Helium atom is 2.2 times the energy required to remove an electron from Helium atom. The total energy required to ionize the Helium atom completely is :

A. 20 eV
B. 34 eV
C. 79 eV
D. 109 eV

Answer ||| C

Solution ||| Energy required to remove e^- from singly ionized helium atom = 54.4 eV

Let energy required to remove e^- form helium atom = N eV
From the question,
54.4 eV = 2.2×N
∴ N = 24.72 eV
Energy required to ionize helium atom = (54.4 + 24.72) = 79.12 eV

26.A solution containing active cobalt $^{60}_{27}$Co having activity of 0.8 μCi and decay constant λ is injected in an animal's body. If 1 cm3 of blood is drawn from the animal's body after 10 hrs of injection, the activity found was 300 decays per minute. What is the volume of blood that is flowing in the body ? (1 Ci=3.7×10^{10} decays per second and at t = 10 hrs $e^{-\lambda t}$ = 0.84)

A. 6 liters

B. 7 liters

C. 4 liters

D. 5 liters

Answer ||| D

Solution ||| Let the volume of the blood is v

Initial activity A_0 = 0.8 μCi

Its activity at time t = A = $A_0 \times e^{-\lambda t}$

Activity of x volume will be:

$$A' = \left(\frac{A}{V}\right)x = x\left(\frac{A_0}{V}\right)e^{-\lambda t}$$

$$\therefore V = x\left(\frac{A_0}{A'}\right)e^{-\lambda t}$$

$$\therefore V = (1\ cm^3)\left(\frac{8\times10^{-7}\times3.7\times10^{10}}{\frac{300}{600}}\right)(0.84) = 4.97\times10^3\ cm^3$$

$$= 4.97\ liters$$

27.In a common emitter configuration with suitable bias, it is given that R_L is the load resistance and R_{BE} is small signal dynamic resistance (input side). Then, voltage gain, current gain and power gain are given, respectively, by : β is current gain, I_B, I_C and I_E are respectively base, collector and emitter currents.

A. $\beta\frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_B}, \beta^2\frac{R_L}{R_{BE}}$

B. $\beta\frac{R_L}{R_{BE}}, \frac{\Delta I_E}{\Delta I_B}, \beta^2\frac{R_L}{R_{BE}}$

C. $\beta\frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_E}, \beta^2\frac{R_L}{R_{BE}}$

D. $\beta\frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_B}, \beta\frac{R_L}{R_{BE}}$

Answer ||| A

Solution ||| The question is directly based on the formulae:

$$\text{Current gain } \beta = \frac{\Delta I_C}{\Delta I_B}$$

$$Voltage\ gain\ A_v = \frac{\Delta V_{CE}}{R_{BE}\,\Delta I_B} = \beta\frac{R_L}{R_{BE}}$$

$$Power\ gain\ A_p = \beta A_v = \beta^2\frac{R_L}{R_{BE}}$$

28.The number of amplitude modulated broadcast stations that can be accommodated in a 300 kHz band width for the highest modulating frequency 15 kHz will be :

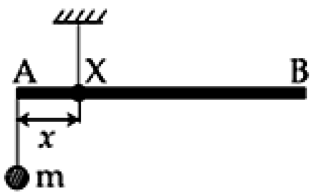
A. 20
B. 15
C. 10
D. 8

Answer ||| C

Solution ||| Bandwidth is twice that of modulating frequency
Thus, if modulating frequency is 15 KHz then bandwidth of one channel = 30 kHz

Now, given band is 300kHz and Bandwidth is 30kHz

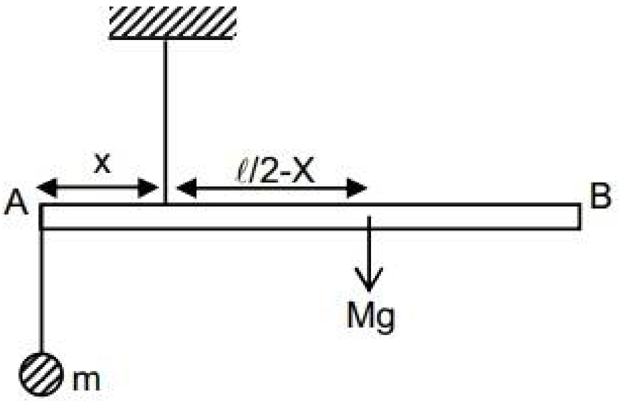
Number of channels that can be accommodated
=Band/bandwidth
= 300 kHz/30 kHz
= 10

29. 

A uniform rod AB is suspended from a point X, at a variable distance x from A, as shown. To make the rod horizontal, a mass m is suspended from its end A. A set of (m, x) values is recorded. The appropriate variables that give a straight line, when plotted are

A. m, x
B. $m, \frac{1}{x}$
C. $m, \frac{1}{x^2}$
D. m, x^2

Answer ||| B

Solution ||| 

To keep the rod horizontal, the body should be balanced about its point of suspension.

Balancing torques at the point of suspension,

$$mgx = Mg\left(\frac{l}{2} - x\right)$$

$$\therefore mx = M\times\frac{l}{2} - Mx$$

$$\therefore m = \left(M\times\frac{l}{2}\right)\left(\frac{1}{x}\right) - M$$

Comparing above equation with y= αx+C (Equation of straight line) we get,

y=m, x= (1/x)

Hence, (y,x) = (m, 1/x)

30.In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of 0.25 cm. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is :

A. 0.4300 cm

B. 0.2150 cm

C. 0.3150 cm

D. 0.0430 cm

Answer ||| B

Solution ||| Least count of the screw gauge

$$= \frac{Pitch\ of\ screw\ gauge}{Total\ number\ of\ divisions\ of\ circular\ scale}$$

Here, pitch = 0.25 cm

And the total circular division = 5 x 100 = 500

Thus,

$$L.C. = \frac{0.25}{5\times100}\ cm = 5\times 10^{-4}\ cm$$

Since 5 complete rotations give 0.25 cm of linear distance.

Thus, 1 complete rotation will give 0.05 cm of linear distance.

Reading is of 4 main scale division and each main scale division moves by 0.05cm.

Thus, Main scale reading = 4 x 0.05 cm

Now, Screw gauge reading = Main scale reading + Division on circular scale x Least Count

$= 4 \times 0.05\text{cm} + 30 \times 5 \times 10^{-4}$ cm

= (0.2 + 0.0150) cm = 0.2150 cm